Score:

Name:______________________________

Section (circle one): 1021 2021
Team (circle one): a b c d e

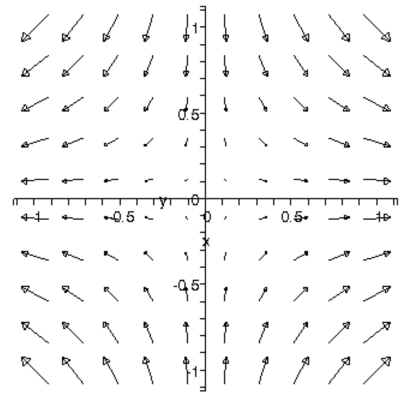
SM221 – Sample Test #4– Fall 2004

Part 1: Multiple Choice (50%). For each question, circle the letter for he best answer.

1. The vector field procured could be the gradient vector field of the function *f*, if $f(x,y) =$

   (a) $\frac{xy}{4}$ (b) $-\frac{xy}{4}$ (c) $\frac{(x^2+y^2)}{8}$

   (d) $\frac{(-x^2+y^2)}{8}$ (e) $\frac{(x^2-y^2)}{8}$



2. The work done by the force field ∇f, where $f(x,y) = (x^2+y^2)^{-\frac{1}{2}}$, on a particle that moves from the point (0, 2) to (3, 4) is:

   (a) $-\frac{\sqrt{481}}{100}$ (b) $-\frac{3}{10}$ (c) $-\frac{\sqrt{13}}{5}$ (d) $-\frac{\sqrt{13}}{2}$ (e) -3

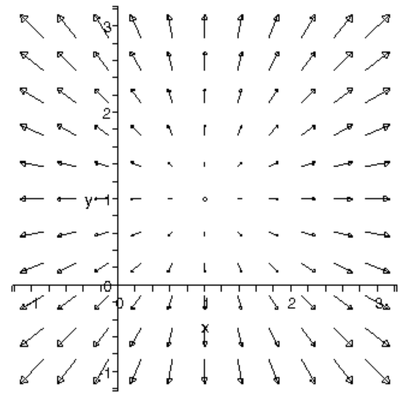
3. Consider a simply closed curve *C* oriented in a counter-clockwise direction in a domain *D*. According to Greens theorem $\oint_C y^2\,dx + x^2\,dy = \iint_D f(x,y)dA$, where $f(x,y) =$

   (a) *x-2y* (b) *2x-2y* (c) 1 (d) *x+2y* (e) *2x+2y*

4. If $\vec{F}(x,y,z) = x^2\vec{i}$ and *S* is the boundary surface of the cube with vertices at (0,0,0), (1,0,0), (1,1,0), (0,1,0), (0,0,1), (1,0,1), (1,1,1), and (0,1,1), then according to the divergence theorem, $\iint_S \vec{F}(x,y,z)\cdot dS =$

   (a) -1 (b) $-\frac{1}{2}$ (c) 0 (d) $\frac{1}{2}$ (e) 1

5. The vector field $\vec{F}(x, y, z)$ is pictured. (Only the portion in the *xy*-plane is shown, but this vector field looks the same in all other planes parallel to the *xy*-plane. The $\vec{k}$ component of all the vectors is 0.) Which of these statements must be true at the point *PI*(1,1,1)?



   (a) $div(\vec{F}) > 0$ and $curl(\vec{F}) = 0$
   (b) $div(\vec{F}) < 0$ and $curl(\vec{F}) = 0$
   (c) $div(\vec{F}) = 0$ and $curl(\vec{F}) = 0$
   (d) $div(\vec{F}) > 0$ and $curl(\vec{F}) \neq 0$
   (e) $div(\vec{F}) < 0$ and $curl(\vec{F}) \neq 0$

---

6. Let *f* be a scalar field, and let $\vec{F}$ be a vector field. Which of the following expressions is meaningful:

(a) $\nabla f + \nabla \cdot f$ (b) $\nabla \vec{F} + \nabla \cdot f$ (c) $\nabla f + \nabla \cdot \vec{F}$ (d) $\nabla f + \nabla \times \vec{F}$ (e) $\nabla \times f + \nabla f$

---

7. Suppose the vector field $\vec{F}$ is conservative. Which of the following statements is necessarily true?
   (a) $\int_C \vec{F} \cdot d\vec{r} = 0$ for any path *C*.
   (b) The divergence of $\vec{F}$ is zero.
   (c) $\vec{F}$ is a constant vector field.
   (d) The flux of $\vec{F}$ through any surface is zero.
   (e) The curl of $\vec{F}$ is zero.

---

8. Let *T* be a triangle with vertices at (0,0), (5,0), and (0,4). If *T* is oriented counterclockwise, the line integral $\int_T (x+3y)dx + (x-y)dy$ is equal to:

   (a) 0 (b) 10 (c) 20 (d) -10 (e) -20

---

9. Let $\vec{F}(x, y, z) = \dfrac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{x^2 + y^2 + z^2}}$. Which of the following is a potential function for $\vec{F}$

   (a) $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ (b) $\sqrt{x^2 + y^2 + z^2}$ (c) $x^2 + y^2 + z^2$
   (d) $\dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$ (e) $\vec{F}$ has no potential.

---

10. Let *C* be the closed rectangular curve with vertices at (1,0,0), (0,2,0), (0,2,1), and (1,0,1), oriented so the vertices go in that order. Then $\int_C (y\vec{i} - x\vec{j} + z\vec{k}) \cdot d\vec{r}$ is

   (a) 0 (b) 2 (c) -2 (d) 6 (e) -6

---

11. The flux of $\vec{F} = 2xy\vec{i} - y^2\vec{j} + 2z\vec{k}$ through a sphere *S* with equation $x^2 + y^2 + z^2 = 9$, computed with an outward normal for *S*, is
   (a) 0 (b) 12π (c) -12π (d) 72π (e) 9π

Name:_______________________________

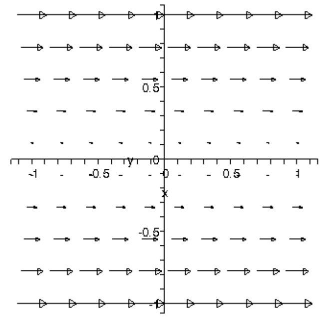
12. A function *f(x,y)*, whose gradient is given by $\vec{F}(x,y) = \langle 2xy, -x^2 \rangle$ is:

(a) $x^2 y$ (b) $\langle x^2 y, -x^2 y \rangle$ (c) 0 (d) $-x^2 y$ (e) does not exist

---

13. The vector field whose picture appears to the right, in general,

(a) has 0 gradient.
(b) has 0 curl and divergence.
(c) has zero curl and non-zero divergence.
(d) has non-zero curl and zero divergence.
(e) has non-zero curl and divergence.



Part 2: Free Response (50 %). The remaining problems are not multiple choice. Answer them in the space below the problem. Show the details of your work and clearly indicate your answers.
(Challenge Factor: *Easy, ***Challenging, *****Very Challenging)

---

14. (**) Use Green's Theorem to compute $\oint_C y^3\,dx - x^3\,dy$ where *C* is the positively oriented curve consisting of a line segment from (-3,0) to (3,0) and the upper half of the circle $x^2 + y^2 = 9$.

---

15. (*)Given $\vec{F}(x,y,z) = \langle x\sin(y), y\cos(x), xyz \rangle$, find

(a) $\nabla \times \vec{F}$
(b) $\nabla \cdot \vec{F}$

---

16. (**)Use the divergence theorem to find $\iint_S \vec{F} \cdot d\vec{S}$ if $\vec{F}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$ and *S*, the surface of a sphere of radius 1 centered on the origin.

---

17. (*)Let $f(x,y) = x^2 + y^2 + x^2 y + 4$.

(a) Compute $\nabla f(x,y)$.
(b) Compute $\nabla \cdot \nabla f(x,y)$.
(c) Compute $\nabla \times \nabla f(x,y)$.

---

18. (***)Let $\vec{F}(x,y) = (xy - y^2)\vec{i} + (x^2 + 2y)\vec{j}$.

(a) Compute $\int_C \vec{F}(x,y) \cdot d\vec{r}$, where *C* is the line segment from (0,0) to (3,0).
(b) Compute $\int_C \vec{F}(x,y) \cdot d\vec{r}$, where *C* is the line segment from (0,0) to (3,0), followed by the line segment from (3,0) to (3,1).
(c) In view of your answers to (a) and (b), could $\vec{F}$ be conservative? Explain.

---

19. (***) Solve the following:

(a) Find the work done by the force field $\vec{F}(x,y,z)=(xz)\vec{i}+(xy)\vec{j}+(yz)\vec{k}$ on a particle that moves from the origin to (1,1,1) along the curve $\vec{r}(t)=t^2\vec{i}+t^3\vec{j}+t^4\vec{k},\ 0\le t\le 1$.

(b) Would the work be the same of different if the particle followed a different path from the origin to (1,1,1)? How to we know?

---

20. (***) S is the portion of the plane $z=2x+3y$ which lies above the rectangle $0\le x\le 2,\ 0\le y\le 1$, oriented upward (in the direction of the positive z-axis.) $\vec{F}(x,y,z)=(xz)\vec{i}+(xy)\vec{j}+(yz)\vec{k}$. Compute the flux of $\vec{F}$ across S, $\iint \vec{F}\cdot d\vec{S}$.
