

Score:

Name: _____

Section (circle one): 1 2 3 4 5 6

Team (circle one): a b c d e f

SM221 – Sample Test #3

Part 1: Multiple Choice (50%). For each question, circle the letter for the best answer.

1. The volume under the paraboloid $z = x^2 + 2y^2$ and above the region in the xy -plane bounded by the x -axis and y -axis, and the line $x+y=1$ is:

- (a) $\frac{1}{4}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{3}{4}$ (e) 1

2. The volume above the paraboloid $z = x^2 + y^2$ and below the paraboloid $z = 1 - x^2 - y^2$ is:

- (a) 0 (b) $\frac{3}{16}\pi$ (c) $\frac{1}{4}\pi$ (d) $\frac{2}{3}\pi$ (e) $\frac{2\sqrt{2}}{3}\pi$

3. The iterated integral $\int_{-2}^0 \int_0^{-x} f(x, y) dy dx$ must be equal to:

- (a) $\int_0^{-x} \int_{-2}^0 f(x, y) dx dy$ (b) $\int_0^2 \int_{-y}^{-2} f(x, y) dx dy$ (c) $\int_{-2}^0 \int_0^y f(x, y) dx dy$
(d) $\int_0^2 \int_{-2}^{-y} f(x, y) dx dy$ (e) $\int_{-2}^0 \int_{-y}^0 f(x, y) dx dy$

4. ~~The surface area of the portion of the plane $z=2x+3y$ which lies above the rectangle $0 \leq x \leq 2$ and $0 \leq y \leq 1$ is:~~

- ~~(a) $2\sqrt{6}$ (b) $2\sqrt{14}$ (c) 9 (d) 12 (e) 28~~

5. $\int_0^{\pi/2} \int_0^{\sin x} y dy dx =$

- (a) $\frac{1}{2} \sin^2 x$ (b) π (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{8}$ (e) 0

6. The integral $f(x, y) = x^2 + y$ over the disk $x^2 + y^2 \leq 1$ is:

- (a) $\int_0^1 \int_0^1 (x^2 + y) dx dy$ (b) $4 \int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y) dx dy$ (c) $\int_0^1 \int_0^{2\pi} (r^2 \cos^2 \theta + r \sin \theta) r d\theta dr$
(d) 0 (e) $\int_0^1 \int_0^{2\pi} (\cos^2 \theta + \sin \theta) d\theta dr$

7. Let R be the region of a 3-space bounded by the xy -plane, the surface $x^2 + y^2 = 1$, and the plane $y+z=1$. Then $\iiint_R k dV =$

- (a) $\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 z dx dy dz$ (b) $\int_{-1}^1 \int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{1-y} z dz dy dx$ (c) $\int_{-1}^1 \int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} (1-y) dy dx$
(d) $\int_0^1 \int_{-1}^1 \int_{-1}^{1-y} z dx dy dz$ (e) $\int_0^1 \int_0^{2\pi} (1-r \sin \theta) r dr d\theta$

8. If we reverse the order of integration in the integral $\int_0^1 \int_0^{x^2} \sin(y) dy dx$, we obtain:

- (a) $\int_0^1 \int_{\sqrt{y}}^1 \sin(y) dx dy$ (b) $\int_0^1 \int_{\sqrt{y}}^1 \sin^{-1}(x) dx dy$ (c) $\int_0^1 \int_{y^2}^0 \sin(y) dx dy$ (d) $\int_0^1 \int_0^{y^2} \sin(y) dx dy$
(e) $\int_0^1 \int_x^1 \cos(y) dx dy$
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9. When we convert the iterated integral $\int_0^3 \int_0^{\sqrt{9-x^2}} e^{x^2+y^2} dy dx$ from rectangular to polar coordinates, we obtain:

- (a) $\int_0^{\pi/2} \int_0^3 e^{r^2} r dr d\theta$ (b) $\int_0^{2\pi} \int_0^3 e^{r^2} r dr d\theta$ (c) $\int_0^{\pi/2} \int_0^3 r e^{r^2} r dr d\theta$ (d) $\int_0^3 \int_0^{\sqrt{9-(r\cos\theta)^2}} e^{r^2} r dr d\theta$
(e) $\int_0^3 \int_0^{\sqrt{9-(r\cos\theta)^2}} e^{r^2} r dr d\theta$
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10. Let R be the rectangular region with vertices $(0,0)$, $(1,0)$, $(1,3)$, and $(0,3)$. Then $\iint_R 223y^2 dA =$
(a) 0 (b) 3 (c) 223 (d) 669 (e) 2007

11. The region of integration for the iterated integral $\int_0^4 \int_0^{6-x} (x^2 + y^2) dy dx$ is a:

- (a) quarter-circular disk (b) circular disk (c) trapezoid (d) right triangle (e) rectangle
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12. The iterated integral $\int_0^1 \int_0^x f(x, y) dy dx$ is equivalent to:

- (a) $\int_0^1 \int_y^1 f(x, y) dx dy$ (b) $\int_0^1 \int_1^y f(x, y) dx dy$ (c) $\int_0^1 \int_0^1 f(x, y) dx dy$ (d) $\int_0^1 \int_0^y f(x, y) dx dy$
(e) $\int_0^{\pi/4} \int_0^1 f(r \cos \theta, r \sin \theta) r dr d\theta$
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13. Evaluate $\iiint_E (x^2 + y^2 + z^2) dV$ where E is the solid unit sphere centered at the origin.

- (a) $\frac{2}{5}\pi$ (b) $\frac{2}{3}\pi$ (c) $\frac{4}{5}\pi$ (d) π (e) $\frac{4}{3}\pi$
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14. A solid is bounded above by the surface $z = x^2 + y^2$ and below by the region in the xy -plane bounded by $y=0$ and $y=1-x^2$. If the density is given by $\rho(x, y, z) = z$, the mass of the solid is best computed with:

- (a) A double integral in rectangular coordinates.
(b) A double integral in polar coordinates.
(c) A triple integral in rectangular coordinates.
(d) A triple integral in cylindrical coordinates.
(e) A triple integral in spherical coordinates.
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15. The volume of the solid below the surface $z = 16 - x^2 - y^2$ and above the xy -plane is:

- (a) $-\frac{128}{3}\pi$ (b) $\frac{128}{3}\pi$ (c) $\frac{256}{3}\pi$ (d) 128π (e) 256π

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16. The point on the Cartesian coordinates $(x,y,z)=(2,2,0)$ has the spherical coordinates (ρ,θ,φ)
=

- (a) $\left(\sqrt{8}, \frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left(8, \frac{\pi}{4}, \frac{\pi}{2}\right)$ (c) $\left(\sqrt{8}, \frac{\pi}{2}, \frac{\pi}{4}\right)$ (d) $\left(8, \frac{\pi}{4}, \frac{\pi}{2}\right)$ (e) $\left(8, \frac{\pi}{2}, \frac{\pi}{2}\right)$
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Part 2: Free Response (50 %). The remaining problems are not multiple choice. Answer them in the space below the problem. Show the details of your work and clearly indicate your answers.

(Challenge Factor: *Easy, *Challenging, *****Very Challenging)**

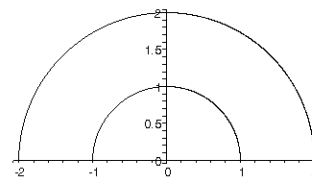
1. (****) For the double integral $\int_0^1 \int_x^{1+3x} xy \, dy \, dx$,
- (b) Make a careful sketch of the region of integration, labeling the coordinates of all corners.
(c) Rewrite the integral for the opposite order of the variables.
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2. (**) Let R be the portion of the xy -plane with $1 \leq x^2 + y^2 \leq 9$.
- (d) Sketch R , and use your sketch to explain why it is sensible to set up an integral over R in a coordinate system that is not Cartesian.
(e) Compute $\iint_R e^{x^2+y^2} \, dA$.
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3. (***) For the triple integral $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} (x^2 + y^2 + z^2) \, dz \, dy \, dx$
- (a) Draw the region of integration.
(b) Rewrite the integral in cylindrical coordinates.
(c) Rewrite the integral in spherical coordinates.
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4. (**) Determine the volume of a tetrahedron bounded the planes $x=0$, $y=0$, $z=0$ and $3x+2y+z=6$.
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5. (**) Find $\iint_D \sqrt{x^2 + y^2} \, dA$ where D is the region shown:



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6. (***) Find the volume of the solid bounded above by the plane $z=x+y$ and below by the region in the xy plane bounded by $y=0$ and $y = 2x - x^2$.
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7. (***) Find the mass of a solid cylinder of radius 5 and height 3, with its base sitting on the xy -plane with the center of the base at the origin if the density is given by $\rho(x, y, z) = x^2 + y^2$.
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8. (***) A lamina (a two dimensional object) occupies the region in the xy -plane bounded by the curves $y=x$ and $y = x^2$. Find the center of mass of the lamina if the density at the point (x, y) is $\rho(x, y) = 2y$.
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9. (***) Find the mass of the tetrahedron bound by the coordinate planes and the plane $3x+2y+z=6$, if the density at the point (x, y, z) is $\rho(x, y, z) = x^2 + z$. Set up the integral, then solve with your calculator!!!
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10. (*****) E is the solid bounded below by the cone $z^2 = x^2 + y^2$ and bounded above by the sphere $x^2 + y^2 + z^2 = 8$. The density at the point (x, y, z) is $\rho(x, y, z) = z$.

- (f) Set up an iterated integral in cylindrical coordinates which gives the mass of E .
- (g) Set up an iterated integral in spherical coordinates which gives the mass of E .
- (h) Evaluate at least one of the integrals.
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