SM221 – Sample Test #1

Part 1: Multiple Choice (50%). For each question, circle the letter for he best answer.

1. Let A, B, C, D, and E be the vertices (in order) of a pentagon with each side of length 1. Then $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$ equals:

(a) $\overrightarrow{AD}$ (b) 3 (c) $\overrightarrow{AB}-\overrightarrow{CD}$ (c) $\overrightarrow{DA}$ (d) $\overrightarrow{AE}$ (e) $\overrightarrow{AB}+\overrightarrow{CD}$

2. If $|y|=4$, $|z|=5$, and $y \cdot z = 0$, $|y \times z|$ is:

(a) 0 (b) $\sqrt{10}$ (c) $\sqrt{20}$ (d) 10 (e) 20

3. The line through the two points (-1,2,1) and (1,1,2) also contains the point:

(a) (0,0,0) (b) (0,3,3) (c) (3,0,3) (d) (3,3,0) (e) (3,2,3)

4. The angle between the vectors $\langle 2,-2,1\rangle$ and $\langle 3,0,0\rangle$ is approximately:

(a) 0.383 rad (b) 0.841 rad (c) 0.931 rad (d) 6 rad (e) 48.2 rad

5. The plot for the equation $x^2+4y^2+9z^2=36$ is a:

(a) sphere (b) cylinder (c) ellipsoid (d) parabolic cylinder (e) plane

6. The equation of the line through the point (1,3,-1) perpendicular to the plane $2x-y+z=3$ is given by:

(a) $x=1+2t$, $y=3-t$, $z=-1+t$

(b) $x=2+t$, $y=-1+3t$, $z=1-t$

(c) $x=-1+2t$, $y=-3-t$, $z=-1-t$

(d) $x=-2t$, $y=t$, $z=-t$

(e) $x=2-t$, $y=-1-3t$, $z=1+t$

7. Which of these planes is <u>parallel</u> to the line $x=2-t,\ y=-2+\frac{1}{2}t,\ z=1+2t$?

(a) $x-\frac{1}{2}y-2z=2007$ (b) $2x-2y+z=2007$ (c) $x-2y-\frac{1}{2}z=2007$

(d) $-\frac{1}{2}x+\frac{1}{2}y-z=2007$ (e) $2x+z=2007$

8. Which of these planes is <u>perpendicular</u> to the line $x=2-t,\ y=-2+\frac{1}{2}t,\ z=1+2t$?

(a) $x-\frac{1}{2}y-2z=2007$ (b) $2x-2y+z=2007$ (c) $x-2y-\frac{1}{2}z=2007$

(d) $-\frac{1}{2}x+\frac{1}{2}y-z=2007$ (e) $2x+z=2007$

9. Suppose $\vec{u}$ and $\vec{w}$ are unit vectors, and the angle between them is $30°$. What is the magnitude of $|\vec{u}\times\vec{w}|$?

(a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\frac{1}{2}$ (e) $\frac{\sqrt{3}}{2}$

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10. Suppose $\vec{v}$ and $\vec{w}$ are vectors. Which of the following expressions is a vector?

(a) $\vec{v}\cdot\vec{w}$ (b) $|\vec{v}|+\vec{w}$ (c) $\vec{v}/\vec{w}$ (d) $|\vec{v}|\vec{w}$ (e) $|\vec{v}+\vec{w}|$

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11. If $\vec{v}=\langle 0,2,-1\rangle$ and *P* is the point (0,2,-1), then $0(x-4)+2(y-1)-(z-2)=0$ is the equation of:

(a) a line parallel to $\vec{v}$ (b) a line through *P* (c) a plane parallel to $\vec{v}$

(d) a planed through *P* (e) a plane perpendicular to $\vec{v}$

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12. Which of the following is a unit vector?

(a) $\langle 2,1,-2\rangle$ (b) $\left\langle \frac{2}{3},\frac{1}{3},-\frac{2}{3}\right\rangle$ (c) $\langle 1,1,1\rangle$ (d) $\langle 3,3,3\rangle$ (e) $\left\langle \frac{1}{3},\frac{1}{3},\frac{1}{3}\right\rangle$

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13. The vertices of a rectangle are A, B, C, and D (in order clockwise). The vector $\overrightarrow{AB}-\overrightarrow{BC}$ is equal to:

(a) $\overrightarrow{AC}$ (b) $\overrightarrow{DB}$ (c) $\overrightarrow{AD}$ (d) $\overrightarrow{BC}$ (e) $\overrightarrow{CB}$

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14. A vector perpendicular to both $\langle 1,2,3\rangle$ and $\langle 2,1,-1\rangle$ is:

(a) $\langle -5,7,-3\rangle$ (b) $\langle -2,1,0\rangle$ (c) $\langle 0,-3,2\rangle$ (d) $\langle 3,3,3\rangle$ (e) $\langle 0,1,0\rangle$

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15. Suppose $\vec{r}\,'(t)=\langle 4e^{2t},4t,\cos(t)\rangle$ and $\vec{r}(0)=\langle 2,1,0\rangle$, then $\vec{r}(1)$ equals:

(a) $\langle 2e^2,3,\sin(1)\rangle$ (b) $\langle 2e^2+2,2,0\rangle$ (c) $\langle 2e^2,2,\sin(1)\rangle$ (d) $\langle 8e^2,3,\sin(1)\rangle$ (e) $\langle 8e^2,4,-\sin(1)\rangle$

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16. A particle's position is given by $\vec{r}(t)=\langle \sin(2t),e^t-1,t^2\rangle$. The particles speed at time *t=0* is:

(a) 0 (b) 2 (c) $\sqrt{5}$ (d) $\sqrt{1+e^2}$ (e) $\sqrt{4+e^2}$

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17. If $\vec{r}(t)=\left\langle t,\frac{1}{\sqrt{2}}t^2,\frac{t^3}{3}\right\rangle$, then the length of the curve between *t=0* and *t=1* is:

(a) 0 (b) 4/3 (c) 7/3 (d) 10/3 (e) 4

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18. If $\bar{r}(t)=\langle t,t^2,t^3\rangle$ then a tangent vector to the curve at the point (1,1,1) is:

(a) $\langle 0, 1, 1\rangle$ (b) $\langle 1, 1, 1\rangle$ (c) $\langle 0, 2, 3\rangle$ (d) $\langle 1, 2, 3\rangle$ (e) $\langle 0, 2, 6\rangle$

19. The length of the curve $\langle 2t, \sin(t), \cos(t)\rangle$ for $0 \le t \le \pi$ is closest to:

(a) 2.24 (b) 3.14 (c) 6.59 (d) 7.02 (e) 10.63

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20. The position of a particle is $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$. Its acceleration when *t=3* is *a(3)=*

(a) $2\vec{j} + 18\vec{k}$ (b) $\vec{i} + 6\vec{j} - 27\vec{k}$ (c) $3\vec{i} + 9\vec{j} - 27\vec{k}$
(d) $4.5\vec{i} + 6.75\vec{j} - 12.15\vec{k}$ (e) $4.5\vec{i} + 9\vec{j} - 20.25\vec{k}$

Part 2: Free Response (50 %). The remaining problems are not multiple choice. Answer them in the space below the problem. Show the details of your work and clearly indicate your answers.

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21. Given the vectors $\vec{u} = \langle 4,3,-12\rangle$ and $\vec{v} = \langle -2,1,2\rangle$ find

(a) $2\vec{u} - 3\vec{v}$ (b) $\vec{u} \cdot 3\vec{v}$ (c) $2\vec{u} \times 3\vec{v}$
(d) a unit vector in the direction of $2\vec{u} - 3\vec{v}$
(e) $comp_{\vec{v}}\,\vec{u}$

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22. A baseball player hits a baseball when it is 1 foot above home plate. He imparts an initial speed of 120 feet per second to the baseball, and it leaves his bat at an angle of $30°$ above the horizontal. Neglecting air resistance and assuming the acceleration due to gravity is 32 feet per second per second, will the baseball pass above a 12 foot high fence 360 feet away.

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23. The position of a particle in the plane is given by $\vec{r}(t) = \langle 6\cos(t/2), 8\sin(t/2)\rangle$ for $0 \le t \le 2\pi$.

(a) Compute the position vector $\vec{r}(\pi)$.
(b) Compute the position vector $\vec{r}\,'(\pi)$.
(c) Compute the position vector $\vec{r}\,''(\pi)$.
(d) Compute the speed at $t = \pi$.

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24. A projectile is fired from the top of a cliff that is 55 meters high over the level sea on a planet where the acceleration due to gravity is 10 *m/sec²* (directed downward). The initial speed of the projectile is 100 *m/sec*, and the angle of inclination is $30°$ above horizontal.

(a) Show that the projectile is airborne for 10.0 *seconds* before hitting the sea.
(b) What is the speed of the projectile at impact?

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25. (a) Find the parametric equations for the line through (1,-1,0) and (2,2,1).
(b) Find the equation of the plane through (1,2,3), (2,5,4), and (0,4,-1).
(c) Verify that your line and your plane are parallel.
(d) Find the distance between any point on your line and your plane.

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26. Consider the points $A=(5,0,0)$, $B=(0,3,0)$, and $C=(0,0,2)$ which are the vertices of a triangle:

(e) Compute $\overrightarrow{CA} \cdot \overrightarrow{CB}$
(f) Determine angle C to the nearest degree.
(g) Find $proj_{\overrightarrow{CA}} \overrightarrow{CB}$
(h) Compute $\overrightarrow{CA} \times \overrightarrow{CB}$.
(i) Find the equation of the plane E that contains A, B, and C.
(j) Find the line through the origin perpendicular to the plane E.
(k) Find the area of the triangle formed by A, B, and C.

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27. A stone is swung around on a string so that the position (as measured in *meters*) of the stone at time t (in *seconds*) is $\vec{r}(t) = 2\cos(t)\vec{i} + (5 + 2\sin(t))\vec{k}$.

(a) Find the velocity, acceleration and speed of the stone at $t=0$.

(b) At $t = \frac{5\pi}{3}$ *seconds*, the string breaks and the stone is only subject to gravity. Find the position $\vec{r}(u)$ of the stone as a function of the number of seconds $u = t - \frac{5\pi}{3}$ after the string breaks, for $u > 0$. (Hint: $g = 9.8 m/s^2$)

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28. Somewhere in the South Pacific: Your ship is traveling on a course 060 at a speed of 10 knots. There is a westerly ocean current with a direction of 270 and a speed of 4 knots. What is your true course and speed?

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29. Somewhere over the North Atlantic: Your F18 Hornet is flying on a course 045 at a speed of 400 knots in the jet stream whose direction is 090 and a speed of 100 knots. What is your true course and ground speed?
