Solutions

to

Sample Test I

Multiple Choice

1) (a) $\vec{AB} + \vec{BC} = \vec{AC} \Rightarrow \vec{AC} + \vec{CD} = \underline{\underline{\vec{AD}}}$

2) (e) $y \cdot z = 0 \Rightarrow$ angle between y & z is $90°$

$\Rightarrow |y \times z| = |y||z| \sin(90°) = 4 \cdot 5 \cdot 1 = 20$

3) (c) Equation for line $\langle x, y, z \rangle = \langle -1, 2, 1 \rangle + \langle 2, -1, 1 \rangle t$

$\Rightarrow x = 0 \Rightarrow t = 1/2 \Rightarrow \langle x, y, z \rangle = \langle 0, 3/2, 3/2 \rangle$ $\cancel{a}$, $\cancel{b}$

$\Rightarrow x = 3 \Rightarrow t = 2 \Rightarrow \langle x, y, z \rangle = \langle 3, 0, 3 \rangle \Rightarrow$ (c)

4) (b) $\cos\theta = \dfrac{\langle 2, -2, 1 \rangle \cdot \langle 3, 0, 0 \rangle}{|\langle 2, -2, 1 \rangle| |\langle 3, 0, 0 \rangle|} = \dfrac{6}{(3)(3)} = 2/3$

$\Rightarrow \theta = \cos^{-1} \frac{2}{3} = .841$ rad (b)

5) (c) - ellipsoid

6) (a) $\langle x, y, z \rangle = \langle 1, 3, -1 \rangle + \langle 2, -1, 1 \rangle t$

$\Rightarrow x = 1 + 2t$
$y = 3 - t$ (a)
$z = -1 + t$

17. (e) Direction of line is $\langle -1, \frac{1}{2}, 2 \rangle$

~~(a)~~ normal to plane $= \langle 1, -\frac{1}{2}, -2 \rangle$

$\Rightarrow \langle -1, \frac{1}{2}, 2 \rangle \cdot \langle 1, -\frac{1}{2}, 2 \rangle = -1 - \frac{1}{4} + 4 \neq 0$

~~(b)~~ normal to plane $= \langle 2, -2, 1 \rangle$

$\Rightarrow \langle -1, \frac{1}{2}, 2 \rangle \cdot \langle 2, -2, 1 \rangle = -2 - 1 + 2 \neq 0$

~~(c)~~ normal to plane $= \langle 1, -2, -\frac{1}{2} \rangle$

$\langle -1, \frac{1}{2}, 2 \rangle \cdot \langle 1, -2, -\frac{1}{2} \rangle = -1 - 1 + 1 \neq 0$

~~(d)~~ normal to plane $= \langle -\frac{1}{2}, +\frac{1}{2}, -1 \rangle$

$\langle -1, \frac{1}{2}, 2 \rangle \cdot \langle -\frac{1}{2}, +\frac{1}{2}, -1 \rangle = \frac{1}{2} + \frac{1}{4} - 2 \neq 0$

(e) normal to plane $= \langle 2, 0, 1 \rangle$

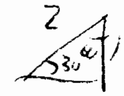
$\Rightarrow \langle -1, \frac{1}{2}, 2 \rangle \cdot \langle 2, 0, 1 \rangle = 0$ ✓

8) (a) Direction of line $\langle -1, \frac{1}{2}, 2 \rangle$

$\Rightarrow$ this is normal to plane

$\Rightarrow -x + \frac{1}{2}y + 2z = d$ or $x - \frac{1}{2}y - 2z = d$

(a)



⑨ ⓐ $|\vec{u} \times \vec{w}| = |\vec{u}||\vec{w}| \sin 30 = (1)(1)(\frac{1}{2}) = \frac{1}{2}$

↑ unit vectors

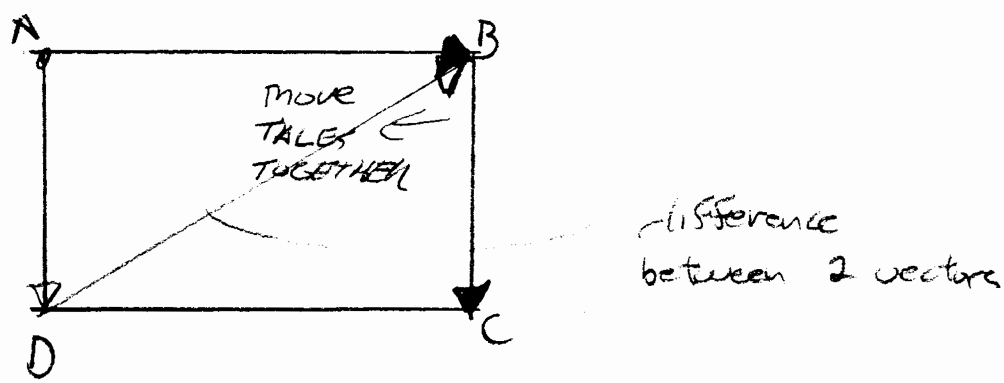
⑩ ⓓ a) ⇒ scalar
b) ⇒ impossible to do
c) undefined function
ⓓ ⇒ vector!! ⟸

⑪ ⓔ Plane perpendicular to $\vec{v}$

⑫ ⓑ $\left|\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle\right| = \left(\frac{4}{9} + \frac{1}{9} + \frac{4}{9}\right)^{\frac{1}{2}} = 1$ ✓

⑬ $\overrightarrow{DB}$

ⓑ



⑭ ⓐ $\langle 1, 2, 3 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 1 & -1 \end{vmatrix}$

$= i \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} - j \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -5i + 7j - 3k$

$= \langle -5, 7, -3 \rangle$

(15) $\vec{r}(t) = \int r'(t)\,dt = \left\langle \int 4e^{2t}\,dt,\ \int 4t\,dt,\ \int \cos(t)\,dt \right\rangle$

$= \langle 2e^{2t} + c_1,\ 2t^2 + c_2,\ \sin(t) + c_3 \rangle$

$\Rightarrow \vec{r}(0) = \langle 2 + c_1, c_2, c_3 \rangle = \langle 2, 1, 0 \rangle$

$\Rightarrow c_1 = 0,\ c_2 = 1,\ c_3 = 0$

$\Rightarrow \vec{r}(t) = \langle 2e^{2t},\ 2t^2 + 1,\ \sin(t) \rangle$

$\Rightarrow \vec{r}(1) = \langle 2e^2,\ 3,\ \sin(1) \rangle$ (a)

(16) $v(t) = r'(t) = \langle 2\cos(2t),\ e^t,\ 2t \rangle$

$\Rightarrow \vec{v}(0) = \langle 2, 1, 0 \rangle \Rightarrow$ speed at $t=0$

$= |v(0)| = \sqrt{2^2 + 1^2 + 0^2} = \sqrt{5}$

(c)

(17) $L = \int_a^b |r'(t)|\,dt$

$\Rightarrow r'(t) = (1, \sqrt{2}t, t^2) \Rightarrow |r'(t)| = \sqrt{1 + 2t^2 + t^4}$

$= \sqrt{(t^2+1)^2} = (t^2 + 1)$

$\Rightarrow L = \int_0^1 (t^2 + 1)\,dt = \frac{1}{3}t^3 + t \Big|_0^1 = 4/3$

(b)

(18) $r'(t) = \langle 1, 2t, 3t^2 \rangle$

$r(t) = \langle 1, 1, 1 \rangle \Rightarrow t = 1$

$\therefore r'(1) = \langle 1, 2, 3 \rangle$ (d)

(19) $r'(t) = \langle 2, \cos(t), -\sin(t) \rangle$

$|r'(t)| = (4 + \cos^2(t) + \sin^2(t))^{1/2} = \sqrt{5}$

$\Rightarrow L = \int_0^{\pi} \sqrt{5}\, dt = \sqrt{5}\, t \Big|_0^{\pi} = (\sqrt{5})(\pi) \approx 7.02$

(d)

(20) $r'(t) = \langle 1, 2t, 3t^2 \rangle$

$\vec{a}(t) = r''(t) = \langle 0, 2, 6t \rangle$

$\Rightarrow \vec{a}(3) = \langle 0, 2, 18 \rangle = 2\vec{j} + 18\vec{k}$

(a)

(15) (a) $\rho = (2^2 + 2^2 + 0)^{1/2} = \sqrt{8}$

$\theta = \tan^{-1} \frac{2}{2} = \tan^{-1}(1) = \pi/4$

$\phi = \cos^{-1} \frac{z}{\rho} = \cos^{-1} 0 = \pi/2$

(c)

Long Answer

21 (a) $\Rightarrow \; 2\vec{u} - 3\vec{v} = \langle 8+6, 6-3, -24-6 \rangle = \underline{\langle 14, 3, -30 \rangle}$

(b) $u \cdot 3v = \langle 4, 3, -12 \rangle \cdot \langle -6, 3, 6 \rangle$

$= -24 + 9 - 72 = \underline{-87}$

(c) $\langle 8, 6, -24 \rangle \times \langle -6, 3, 6 \rangle$

$$\Rightarrow \begin{vmatrix} i & j & k \\ 8 & 6 & -24 \\ -6 & 3 & 6 \end{vmatrix} = i \begin{vmatrix} 6 & -24 \\ 3 & 6 \end{vmatrix} - j \begin{vmatrix} 8 & -24 \\ -6 & 6 \end{vmatrix} + k \begin{vmatrix} 8 & 6 \\ -6 & 3 \end{vmatrix}$$

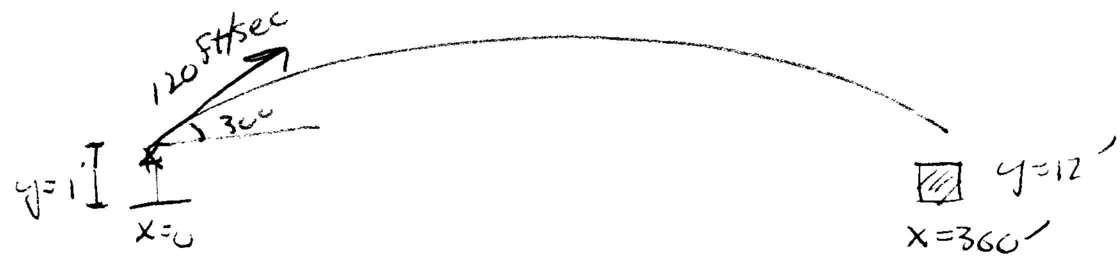
$= 108\vec{i} - 96\vec{j} + 60\vec{k} = \underline{\langle 108, -96, 60 \rangle}$

(d) $\dfrac{\langle 14, 3, -30 \rangle}{(14^2 + 3^2 + 30^2)^{1/2}} = \dfrac{\langle 14, 3, -30 \rangle}{\sqrt{1105}}$

(21) e) $\text{comp}_{\vec{v}}\, \vec{u} = |\vec{u}| \cos\theta = |\vec{u}| \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$

$$= \frac{\langle 4, 3, -12\rangle \cdot \langle -2, 1, 2\rangle}{(2^2+1^2+2^2)^{1/3}} = \frac{-8+3-24}{3} = -\frac{29}{3}$$

(22)



$\vec{a} = \langle 0, -32\rangle$

$\vec{v}(0) = \langle 120\cos 30°, 120 \sin 30°\rangle$
$= \langle 60\sqrt{3}, 60\rangle$

$\vec{r}(0) = \langle 0, 1\rangle$

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle C_1, -32t + C_2\rangle$

$\Rightarrow \vec{v}(0) = \langle C_1, C_2\rangle = \langle 60\sqrt{3}, 60\rangle$
$\Rightarrow \vec{v}(t) = \langle 60\sqrt{3}, 60-32t\rangle$

$\Rightarrow \vec{r}(t) = \int \vec{v}(t)\,dt = \langle 60\sqrt{3}t + C_1, 60t - 16t^2 + C_2\rangle$
$\Rightarrow \vec{r}(0) = \langle C_1, C_2\rangle = \langle 0, 1\rangle$
$\Rightarrow \vec{r}(t) = \langle 60\sqrt{3}t, 60t - 16t^2 + 1\rangle$

$\Rightarrow 60\sqrt{3}t = 360 \Rightarrow t = \frac{6}{\sqrt{3}} = 2\sqrt{3}$

$\Rightarrow y = 120\sqrt{3} - 16(12) + 1 = 16.85''$

CLEARS FENCE! HOME RUN!!

23) a) $\vec{r}(\pi) = \langle 6\cos(\pi/2), 8\sin(\pi/2) \rangle$

$\Rightarrow$ $\underline{\underline{\vec{r}(\pi) = \langle 0, 8 \rangle}}$

b) $r'(t) = \langle -3\sin(t/2), 4\cos(t/2) \rangle$

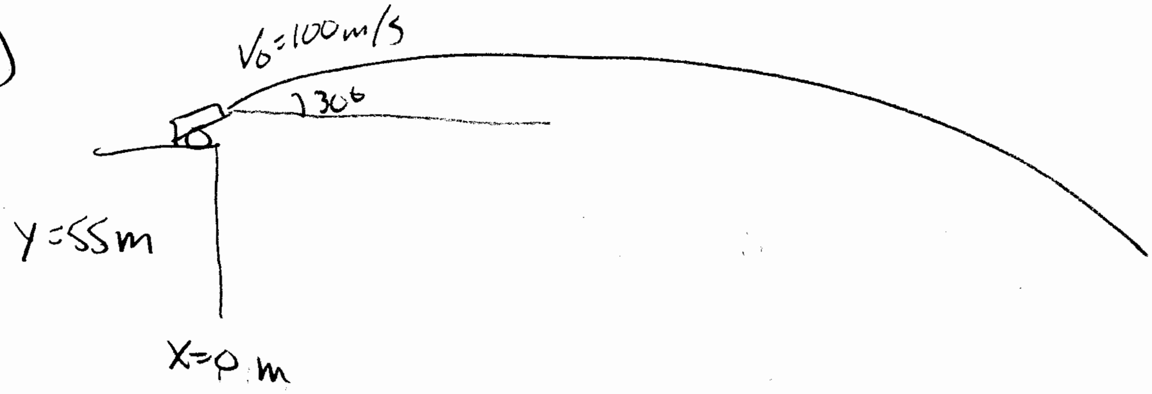
$\Rightarrow$ $\underline{\underline{r'(\pi) = \langle -3, 0 \rangle}}$

c) $r''(t) = \langle -\frac{3}{2}\cos(t/2), -2\sin(t/2) \rangle$

$\underline{\underline{r''(\pi) = \langle 0, -2 \rangle}}$

d) Speed @ $t = \pi = |r'(\pi)| = \sqrt{3^2 + 0^2} = \underline{\underline{3}}$

24)



$\vec{a} = \langle 0, -10 \rangle$

$\vec{V}(0) = \langle 100\cos(30), 100\sin(30) \rangle$
$= \langle 50\sqrt{3}, 50 \rangle$

$\vec{r}(0) = \langle 0, 55 \rangle$

$\Rightarrow \vec{v}(t) = \int \vec{a}(t)\,dt = \langle C_1, -10t + C_2 \rangle$

$\vec{v}(0) = \langle C_1, C_2 \rangle = \langle 50\sqrt{3}, 50 \rangle$

$\Rightarrow \vec{v}(t) = \langle 50\sqrt{3}, 50 - 10t \rangle$

$\Rightarrow \vec{r}(t) = \int \vec{v}(t)\,dt = \langle 50\sqrt{3}t + C_1, 50t - 5t^2 + C_2 \rangle$

$\Rightarrow \vec{r}(0) = \langle C_1, C_2 \rangle = \langle 0, 55 \rangle$

$\Rightarrow \vec{r}(t) = \langle 50\sqrt{3}t, 50t - 5(t)^2 + 55 \rangle$

a) at $t = 10$ $\quad y = 50(10) - 5(10)^2 + 55 = 55$ meters > 0

$\boxed{\text{Therefore has not yet hit water!}}$

b) Hits water when $50t - 5t^2 + 55 = 0 \Rightarrow t^2 - 10t + 11 = 0$

$\Rightarrow (t - 11)(t + 1) = 0 \Rightarrow \underline{\underline{t = 11 \text{ sec}}}$

$\Rightarrow v(11) = \langle 50\sqrt{3}, -60 \rangle \Rightarrow$ speed $= \boxed{105.4\ ft/sec}$

25) a) $\boxed{\begin{aligned} x &= 1+t \\ y &= -1+3t \\ z &= t \end{aligned}}$ ← CAN YOU DO THIS BE INSPECTION ... YES!!

b) Make 2 vectors $\langle 1,3,1\rangle$ $\langle -1,2,-4\rangle$

Take cross product $\begin{vmatrix} i & j & k \\ 1 & 3 & 1 \\ -1 & 2 & -4 \end{vmatrix}$

$\Rightarrow i\begin{vmatrix} 3 & 1 \\ 2 & -4 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ -1 & -4 \end{vmatrix} + k\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix}$

$= -14i + 3j + 5k = \langle -14, 3, 5\rangle$

$\therefore -14x + 3y + 5z + d = 0$

$\Rightarrow$ pick a point to find 'd' i.e. $(0,4,-1)$

$\Rightarrow -14(0) + 3(4) + 5(-1) + d = 0 \Rightarrow d = -7$

$\therefore \underline{\underline{-14x + 3y + 5z - 7 = 0}}$

(c) normal to plane should be perpendical to direction of line.

i.e. $\langle -14, 3, 5\rangle \bullet \langle 1, 3, 1\rangle = -14 + 9 + 5 = 0$ ✓✓

$\therefore$ perpendicular

(d) use $\left|\dfrac{ax_1 + by_1 + cz_1 + d}{(a^2+b^2+c^2)^{1/2}}\right|$ for any point on line i.e. $(+1, -1, 0)$

$$\left|\frac{-14(1) + 3(-1) + 5(0) - 7}{(14^2 + 3^2 + 5^2)^{1/2}}\right| = \left|\frac{-24}{\sqrt{230}}\right| \approx \underline{\underline{1.58251}}$$

26) a) $\vec{CA} = \langle 5, 0, -2\rangle \qquad \vec{CB} = \langle 0, 3, -2\rangle$

$\Rightarrow \vec{CA} \cdot \vec{CB} = 0 + 0 + 4 = \underline{\underline{4}}$

b) $\cos\theta = \dfrac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}||\vec{CB}|} = \dfrac{4}{\sqrt{29}\sqrt{13}} \approx .05714$

$\Rightarrow \underline{\underline{\theta \approx 86.7^\circ}}$

c) $\text{proj}_{\vec{CA}} \vec{CB} = \cancel{|\vec{CB}|} \underbrace{\dfrac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}|\cancel{|\vec{CB}|}}}_{\cos\theta} \dfrac{\vec{CA}}{|\vec{CA}|} = \dfrac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}|^2} \vec{CA}$

$= \underline{\underline{\dfrac{4}{29}\langle 5, 0, -2\rangle}}$

(26)

d) $\vec{CA} \times \vec{CB} = \begin{vmatrix} i & j & k \\ 5 & 0 & -2 \\ 0 & 3 & -2 \end{vmatrix} = i \begin{vmatrix} 0 & -2 \\ 3 & -2 \end{vmatrix} - j \begin{vmatrix} 5 & -2 \\ 0 & -2 \end{vmatrix} + k \begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix}$

$= 6i + 10j + 15k = \underline{\underline{\langle 6, +10, 15 \rangle}}$

e) use $\vec{CA} \times \vec{CB}$ for normal

$\Rightarrow 6x + 10y + 15z + d = 0$

$\Rightarrow$ use $(0,0,2) \Rightarrow 15(2) + d = 0 \Rightarrow d = -30$

$\Rightarrow \underline{\underline{6x + 10y + 15z - 30 = 0}}$

(f) the normal is the direction of the line

$\Rightarrow \langle x, y, z \rangle = \underset{\text{origin}}{\langle 0, 0, 0 \rangle} + \langle 6, 10, 15 \rangle t$

$\Rightarrow \boxed{\begin{aligned} x &= 6t \\ y &= 10t \\ z &= 15t \end{aligned}}$

g) $\text{AREA} = \dfrac{|\vec{CA} \times \vec{CB}|}{2} = \dfrac{(6^2 + 10^2 + 15^2)^{1/2}}{2} = \underline{\underline{\dfrac{19}{2}}}$

27) (a) For $0 \le t \le 5\pi/3$

$\vec{r}(t) = \langle 2\cos t, 0, 5+2\sin(t) \rangle$

$\vec{v}(t) = r'(t) = \langle -2\sin(t), 0, 2\cos(t) \rangle$

$\Rightarrow \boxed{v(0) = \langle 0, 0, 2 \rangle}$ $\boxed{\underset{@\, t=0}{\text{speed}} = |r'(0)| = 2}$

$\Rightarrow \vec{a}(t) = \vec{r}''(t) = \langle -2\cos(t), 0, -2\sin(t) \rangle$

$\Rightarrow \boxed{\vec{a}(0) = \langle -2, 0, 0 \rangle}$

(b) $u = t - 5\pi/3 \Rightarrow$ when $t = 5\pi/3$, $u = 0$

$\vec{V}(t = \frac{5\pi}{3}) = \langle -2\sin(\frac{5\pi}{3}), 0, 2\cos\frac{5\pi}{3} \rangle$

$\Rightarrow \vec{V}(u=0) = \langle \sqrt{3}, 0, 1 \rangle$ <u>Initial Velocity</u>

$\vec{r}(u=0) = \langle 2\cos(5\pi/3), 0, 5+2\sin(\frac{5\pi}{3}) \rangle$

$= \langle 1, 0, 5-\sqrt{3} \rangle$ <u>Initial Position</u>

$\Rightarrow \vec{a}(u) = \langle 0, 0, -9.8 \rangle$

$\Rightarrow \vec{v}(u) = \langle c_1, c_2, -9.8u + c_3 \rangle$

$\Rightarrow \vec{V}(u=0) = \langle c_1, c_2, c_3 \rangle = \langle \sqrt{3}, 0, 1 \rangle$

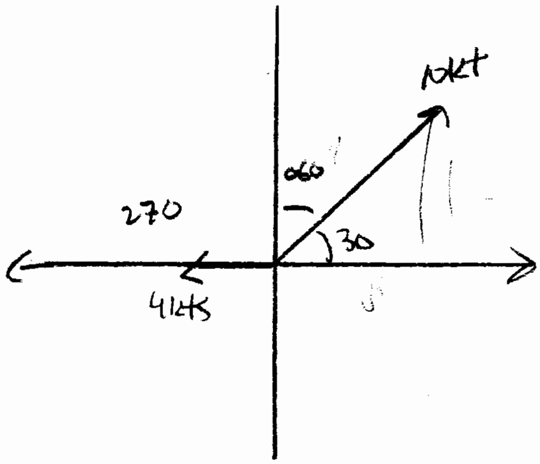
$\therefore \vec{V}(u) = \langle \sqrt{3}, 0, 1 - 9.8u \rangle$

$$\Rightarrow \vec{r}(u) = \int \vec{v}(u)\,du = \langle \sqrt{3}u + C_1,\ C_2,\ u - 4.9u^2 + C_3 \rangle$$

$$\Rightarrow \vec{r}(0) = \langle C_1, C_2, C_3 \rangle = \langle 1, 0, 5-\sqrt{3} \rangle$$

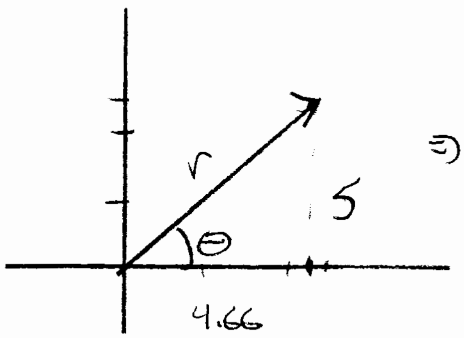
$$\Rightarrow \boxed{\vec{r}(u) = \langle \sqrt{3}u + 1,\ 0,\ u - 4.9u^2 + (5-\sqrt{3}) \rangle}$$

28)



ship vector $= \langle 10\frac{\sqrt{3}}{2}, 10(\frac{1}{2}) \rangle$
wind vector $= \langle -4, 0 \rangle$

$\Rightarrow$ true vector $= \langle 5\sqrt{3}-4, 5+0 \rangle$
$= \langle 4.66, 5 \rangle$



$\Rightarrow$

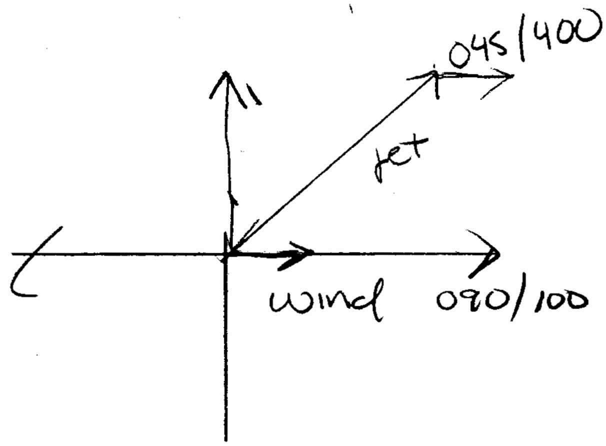
$r = (5^2 + 4.66^2)^{1/2} = 6.84$ kts

$\theta = \tan^{-1}\left(\frac{5}{4.66}\right) = 47°$

$\Rightarrow$ cse $= 90 - \theta = 53°$

$\therefore$ **6.84 kts at 53°**

29)



$$\text{jet} = \langle 400\cos 45^\circ, 400\sin 45^\circ \rangle \text{ knots}$$

$$= \langle 200\sqrt{2}, 200\sqrt{2} \rangle \text{ knots}$$

$$\text{wind} = \langle 100, 0 \rangle$$

$$\Rightarrow \text{true} = \text{jet} + \text{wind} = \langle 200\sqrt{2} + 100, 200\sqrt{2} \rangle$$

$$\approx \langle 382.8, 282.8 \rangle$$

$$\Rightarrow \text{true speed} = (382.8^2 + 282.8^2)^{1/2} \approx 476$$

$$\Rightarrow \text{true cse} = \tan^{-1}\left(\frac{282.8}{382.8}\right) \approx 36.5^\circ$$

$$\therefore \boxed{036.5 / 476 \text{ kts}}$$