SM221, SM221P Calculus 3 FINAL EXAM

Name: ____________ Alpha _____ Instructor ___________ Section: ____

Instructions: No calculator is allowed for Problems 1-21 of this exam. Fill in the top part of your Scantron sheet, including the bubbles for your alpha code. There is no extra penalty for wrong answers in this part. There is room for your work on this exam. Fill in your answers on the bubble sheet. Do Problem 21 in the text booklet, under the problem statement. When you are done with Problems 1-21, hand in your **bubble sheet** and this **exam** to your instructor, who will give you Problems 22-30. You can then use your calculator for Problems 22-30. But you cannot return to Problems 1-21.

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1. Find the length of the vector $\mathbf{a} - \mathbf{b}$ if $\mathbf{a} = \mathbf{i} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j}$.

a. 0
b. $\sqrt{2}$
c. 2
d. $\sqrt{8}$
e. 4

$\langle 1,0,2\rangle - \langle 1,-2,0\rangle = \langle 0,2,2\rangle$

$|\langle 0,2,2\rangle| = \sqrt{8} = 2\sqrt{2}$

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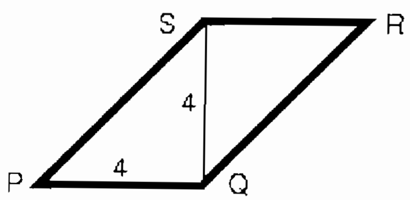
2. The figure shows parallelogram $PQRS$ with base PQ and height QS, both of length 4. What is the magnitude of the cross product $\overrightarrow{PQ} \times \overrightarrow{PS}$?

a. $4\sqrt{2}$
b. 8
c. $8\sqrt{2}$
d. 16
e. $16\sqrt{2}$

= AREA OF PARALLELOGRAM

$= (4)(4) = \boxed{16}$



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3. What is the surface whose equation in cylindrical coordinates is $r^2 + z^2 = 100$?

a. plane
b. cylinder
c. cone
d. circular paraboloid
e. sphere

$= x^2 + y^2 + z^2 = 100$

4. The position of a particle in the plane is $\mathbf{r}(t) = \langle e^{2t}, e^t - 1\rangle$. Find the speed of the particle when it passes through the point $(1,0)$.

a. $\sqrt{5}$
b. $2e^2$
c. $2e^2 + 1$
d. $\sqrt{e^4 + 1}$
e. $\sqrt{4e^4 + 1}$

$\vec{v}(t) = \vec{r}\,'(t) = \langle 2e^{2t}, e^t \rangle$

$\vec{r}(t) = \langle 1, 0 \rangle$ when $t = 0$

$\Rightarrow \vec{v}(0) = \langle 2, 1 \rangle \Rightarrow$ speed $= |\vec{v}(0)| = \sqrt{2^2 + 1^2}$
$= \sqrt{5}$

5. Suppose that $\mathbf{r}'(t) = \langle 2t, 3t^2\rangle$ and $\mathbf{r}(1) = \langle 1,4\rangle$. Find $\mathbf{r}(2)$.

a. $\langle 0,3\rangle$
b. $\langle 2,8\rangle$
c. $\langle 2,10\rangle$
d. $\langle 4,8\rangle$
e. $\langle 4,11\rangle$

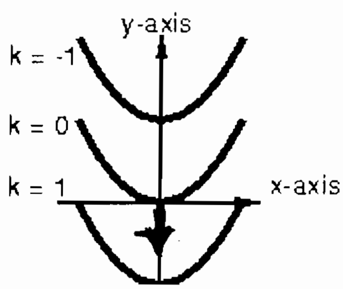
$\vec{r}(t) = \int \vec{r}\,'(t)\,dt = \langle t^2 + C_1, t^3 + C_2 \rangle$

$\Rightarrow \vec{r}(1) = \langle 1 + C_1, 1 + C_2 \rangle = \langle 1, 4 \rangle \Rightarrow C_1 = 0, C_2 = 3$

$\Rightarrow \vec{r}(t) = \langle t^2, t^3 + 3 \rangle \Rightarrow r(2) = \langle 4, 11 \rangle$

6. The contour map shows the level curves $f(x,y) = k$ for $k = -1, 0, 1$. Which vector could be the gradient of f at the origin?

a. $\mathbf{i}$ b. $-\mathbf{i}$ c. $\mathbf{j}$ d. $-\mathbf{j}$ e. $\mathbf{i} + 2\mathbf{j}$



gradient vector is perpendicular to the contour oriented in direction of increase

$\Rightarrow -j$

7. Compute $F_{xy}(2,3)$ if $F(x,y) = x^3y^2$.

a. 6
b. 18
c. 36
d. 48
e. 72

$F_x = 3x^2y^2 \Rightarrow F_{xy} = 6x^2y$

$\Rightarrow F_{xy}(2,3) = 6(2)^2(3) = 72$

8. Let $f(x,y) = e^x \sin(y) + (x-1)^2$. Let $\mathbf{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$. Find the directional derivative of f at the origin in the direction of $\mathbf{u}$.

a. $-\frac{2}{5}$
b. 0
c. $\frac{2}{5}$
d. 1
e. $\frac{7}{5}$

$\nabla f = \langle e^x \sin(y) + 2(x-1), e^x \cos(y) \rangle$

$\nabla f(0,0) = \langle -2, 1 \rangle$

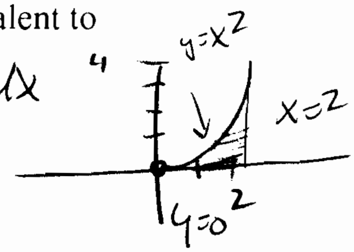
$D_{\vec{u}} f(0,0) = \vec{\nabla} f(0,0) \cdot \vec{u} = \langle -2, 1 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle = -\frac{2}{5}$

9. Reverse the order of integration to obtain a double integral that is equivalent to

$\int_0^4 \int_{\sqrt{y}}^2 f(x,y)\, dx\, dy.$

$x = \sqrt{y}$
$\Rightarrow y = x^2$

$\int_0^2 \int_0^{x^2} f(x,y)\, dy\, dx$



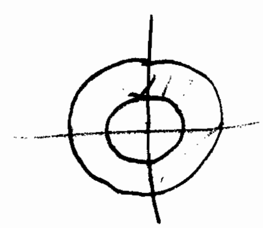
a. $\int_{\sqrt{y}}^2 \int_0^4 f(x,y)\, dy\, dx$ b. $\int_0^2 \int_{x^2}^4 f(x,y)\, dy\, dx$

c. $\int_0^2 \int_0^{x^2} f(x,y)\, dy\, dx$ d. $\int_0^2 \int_0^4 f(x,y)\, dy\, dx$ e. $\int_0^4 \int_{x^2}^2 f(x,y)\, dy\, dx$

10. Let R be the ring-shaped region between the two circles with polar equations $r = 1$ and $r = 2$. Evaluate the double integral

$$\iint_R 12(x^2 + y^2)\, dA.$$

a. 3π
b. 18π
c. 36π
d. 56π
e. 90π



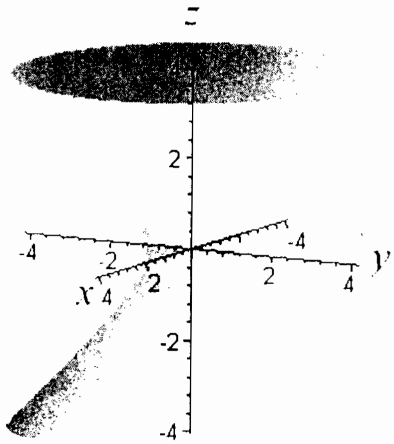
$\int_0^{2\pi} \int_1^2 12 r^2\, r\, dr\, d\theta$

$= 12 \int_0^{2\pi} \int_1^2 r^3\, dr\, d\theta$

$= 3 \int_0^{2\pi} r^4 \Big|_1^2 d\theta = 3 \int_0^{2\pi} (16-1)\, d\theta$

$= 3(15)\, \theta \Big|_0^{2\pi} = 45(2\pi) = 90\pi$

11. Which of the given equations describes the quadric surface graphed below?



a) $x^2 + y^2 - z^2 = 0$ = cone

b) $x^2 + y^2 - z^2 = 1$ = hyperbola 1 sheet

c) $x^2 - y^2 - z^2 = 1$ = hyperbola 2-sheets

~~d)~~ $x^2 + y^2 - z = 0$ paraboloid

e) $x^2 + y^2 + z^2 = 1$ sphere

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12. Which integral gives the length of the curve described by $\mathbf{r}(t) = \left\langle t^2, \sin(t), \cos(2t) \right\rangle$, $0 \le t \le 1$?

~~a)~~ $\int_0^1 (2t + \cos(t) - 2\sin(2t))\,dt$

~~b)~~ $\int_0^1 \sqrt{2t + \cos(t) - 2\sin(2t)}\,dt$

c) $\int_0^1 \left((2t)^2 + \cos^2(t) \cancel{-} (2\sin(2t))^2\right)dt$ no

~~d)~~ $\int_0^1 (2t - \cos(t) + 2\sin(2t))\,dt$

e) $\int_0^1 \sqrt{4t^2 + \cos^2(t) + 4\sin^2(2t)}\,dt$

$\int_0^1 |\vec{r}'(t)|dt$

$\vec{r}'(t) = \langle 2t, \cos(t), -2\sin(2t) \rangle$

$\int_0^1 ((2t)^2 + \cos^2(t) + 4\sin^2(t))^{1/2}dt$

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13. Suppose $z = f(x,y)$, where f is differentiable, $x = g(t)$, $y = h(t)$, and

$$g(3) = 2,\ g'(3) = 5,\ h(3) = -1,\ h'(3) = 2,\ f_x(2,-1) = 4,\ f_y(2,-1) = -2.$$

Using the chain rule, the value of dz/dt when $t = 3$ is:

a) 2
b) 8
c) 10
d) 16
e) 44

$t = 3 \Rightarrow x = 2,\ y = -1$

$\frac{dz}{dt} = \frac{\partial f}{\partial x}\underbrace{\frac{dx}{dt}}_{g'(t)} + \frac{\partial f}{\partial y}\underbrace{\frac{dy}{dt}}_{h'(t)} = (4)(5) + (-2)(2) = 20 - 4 = 16$

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14. Which one of the following iterated integrals finds the mass of the solid hemisphere above the xy-plane and inside the sphere $x^2 + y^2 + z^2 = 4$ if the density at (x,y,z) is z?

a) $\int_0^{\pi/2}\int_0^{2\pi}\int_0^2 \rho^3 \cos(\phi)\sin(\phi)\,d\rho\,d\theta\,d\phi$

b) $\int_0^{\cancel{\pi}}\int_0^{2\pi}\int_0^2 \rho^3 \cos(\phi)\sin(\phi)\,d\rho\,d\theta\,d\phi$

c) $\int_0^{\cancel{\pi}}\int_0^{2\pi}\int_0^2 \rho\cos(\phi)\,d\rho\,d\theta\,d\phi$

d) $\int_0^{\pi/2}\int_0^{2\pi}\int_0^2 \cancel{\rho^2}\cos(\phi)\,d\rho\,d\theta\,d\phi$

e) $\int_0^{2\pi}\int_0^{2\pi}\int_0^1 \cancel{\rho^3}\cos(\phi)\sin(\phi)\,d\rho\,d\theta\,d\phi$

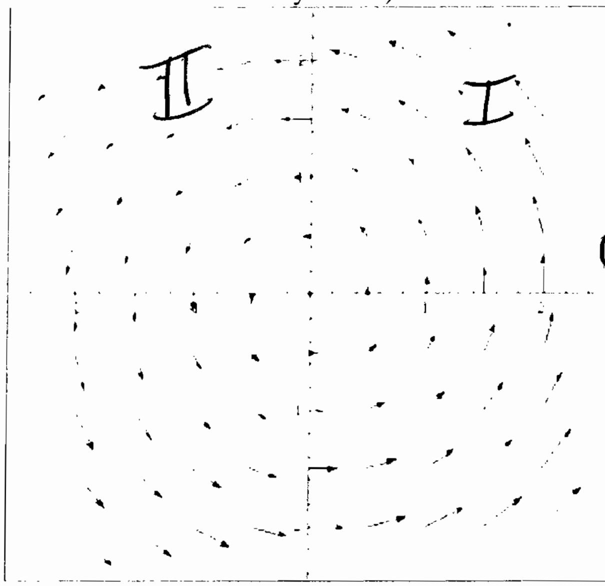
$\int_0^{2\pi}\int_0^{\pi/2}\int_0^2 z\,\rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$ ← hemisphere

$z = \rho\cos\phi$

$\int_0^{2\pi}\int_0^{\pi/2}\int_0^2 \rho^3 \cos\phi\sin\phi\,d\rho\,d\phi\,d\theta$

15. Which vector formula describes the graphed vector field? (As usual, the arrows may have been scaled uniformly to fit.)

In Quadrant I/II



a) $\mathbf{F}(x,y)=\langle y,x\rangle$ $\Rightarrow$ ↗

b) $\mathbf{F}(x,y)=\langle -y,-x\rangle$ $\Rightarrow$ ↙

c) $\mathbf{F}(x,y)=\langle -y,x\rangle$ $\Rightarrow$ ↖ $\langle -,-\rangle$ ↙

$-\ +$

d) $\mathbf{F}(x,y)=\langle y,-x\rangle$ $\Rightarrow$ ↘

$+\ -$

e) $\mathbf{F}(x,y)=\langle -x,y\rangle$ $\Rightarrow$ ↖ $\langle +,+\rangle$ ↗

L u

general direction of arrows

16. Find the work $\int_C \mathbf{F}\cdot d\mathbf{r}$ done by the force $\mathbf{F}(x,y)=\langle x-y,2y\rangle$ where C is the line segment from $(0,0)$ to $(2,3)$.

P Q $\Rightarrow \frac{\partial Q}{\partial x}=0,\ \frac{\partial P}{\partial y}=-1$

a) 0
b) 2
c) 4
d) 6
e) 8

① $x=2t,\ y=3t \Rightarrow \vec{r}(t)=\langle 2t,3t\rangle \quad 0\le t\le 1$

$\Rightarrow$ ② $F=\langle -t,6t\rangle$

③ $r'(t)=\langle 2,3\rangle$

$F\cdot r'(t)=-2t+18t=16t$

$\Rightarrow \int_0^1 16t\,dt = 8t^2\Big|_0^1 = 8$

17. Given the fact that for $f(x,y)=x^2y-y$, $\nabla f=2xy\mathbf{i}+(x^2-1)\mathbf{j}=\mathbf{F}(x,y)$, what is $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is a curve starting at $(3,0)$ and ending at $(2,3)$

a) -18
b) -9
c) 0
d) 9
e) 18

Conservative Force $\vec{F}=\vec{\nabla} f$

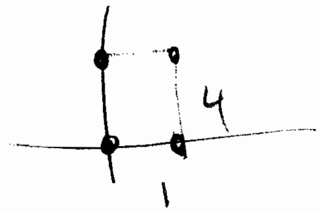
$\Rightarrow \int \vec{F}\cdot d\vec{r} = f(2,3)-f(3,0)$

$=(2)^2(3)-3-[(3)^2(0)-0]$

$=12-3=9$

18. By Green's Theorem, we know that if $Q_x - P_y = 6$ in the rectangular region inside the curve C that consists of segments from (0,0) to (1,0) to (1,4) to (0,4) and back to (0,0), then $\int_C P\,dx + Q\,dy$ equals:

a) 0
b) 4
c) 6
d) 12
e) 24

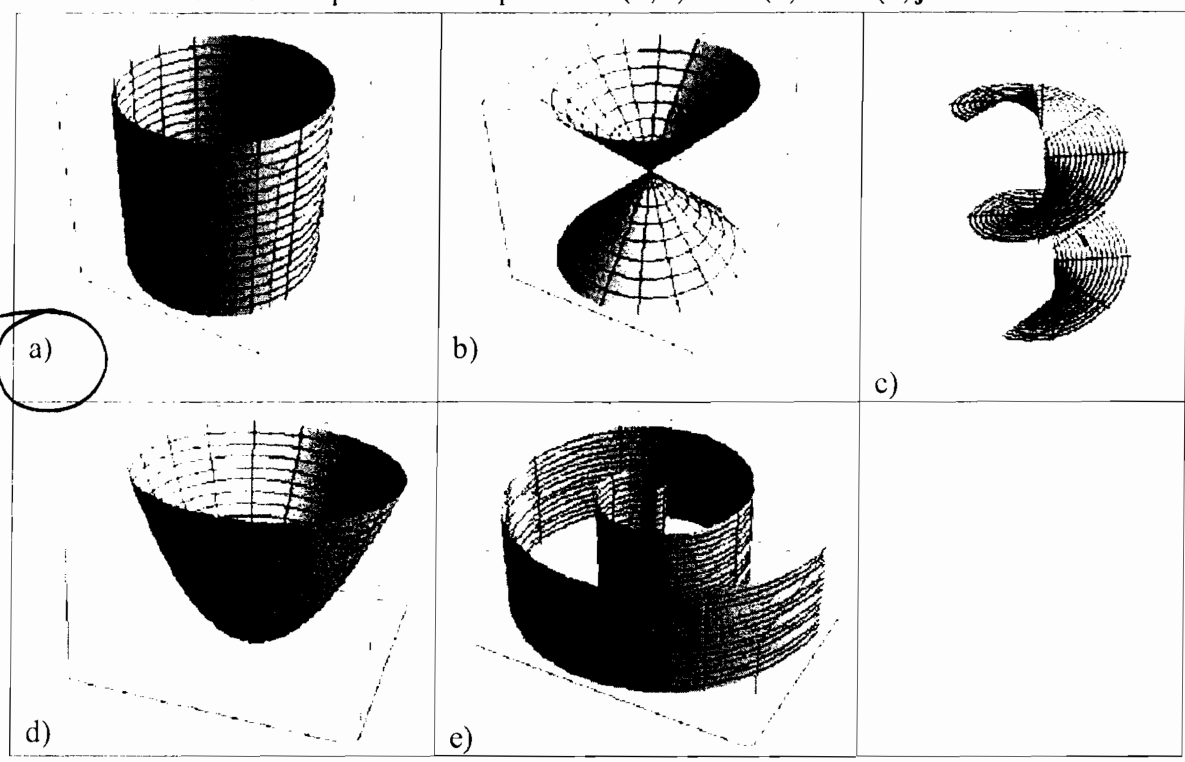


$\iint Q_x - P_y \, dA$

$= 6 \iint dA$ ← area of rectangle

$= (6)(4) = 24$

19. Which surface has parametric equation $\mathbf{r}(u,v) = \cos(v)\mathbf{i} + \sin(v)\mathbf{j} + u\mathbf{k}$?



① x,y has circular motion

② z is independent

⇒ circular cylinder in z-direction

20. For a vector field like $\mathbf{F}(x,y,z) = \langle 2x, x^2 z, y - z \rangle$ for which the divergence is 1, the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ across a positively oriented closed surface S will equal:

a) 0
b) 1
c) The diameter of S.
d) The surface area of S.
e) The volume enclosed by S.

$\iiint \vec{\nabla} \cdot \vec{F} \, dV$

$\uparrow$

1

$= \iiint dV =$ volume

FREE RESPONSE The remaining 10 problems are not multiple choice. Answer Problem 21 on this test paper in the blank space provided. If space is insufficient use the backs of the pages. NO CALCULATOR IS ALLOWED ON Problem 21. After finishing problem 21 turn in all problems 1-21, take out your calculator, and receive and complete Problems 22-30.

21. Prove either of the following Theorems:

a) If $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ is a vector field on $\mathbb{R}^3$ and P, Q, and R have continuous second-order partial derivatives, then $\operatorname{div}(\operatorname{curl}\mathbf{F}) = 0$.

b) If f is a function of three variables that has continuous second-order partial derivatives, then $\operatorname{curl}(\nabla f) = \mathbf{0}$.

(a) (1) $\operatorname{curl}\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = (R_y - Q_z)\vec{i} - (R_x - P_z)\vec{j} + (Q_x - P_y)\vec{k}$

(2) $\vec{\nabla} \cdot \langle R_y - Q_z,\ P_z - R_x,\ Q_x - P_y \rangle =$

$\cancel{R_{yx}} - Q_{zx} + P_{zy} - \cancel{R_{xy}} + Q_{xz} - \cancel{P_{yz}} = 0$

since $R_{yx} = R_{xy}$, etc. (QED)

(b) (1) $\vec{\nabla} f = \langle f_x, f_y, f_z \rangle$

$\operatorname{curl}(\vec{\nabla} f) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$

$= (f_{zy} - f_{yz})\vec{i} - (f_{zx} - f_{xz})\vec{j} + (f_{yx} - f_{xy})\vec{k}$

$= \langle 0, 0, 0 \rangle$ since $f_{zy} = f_{yz}$, ...etc.

QED

When you are done with Problems 1-21, hand in your bubble sheet and the exam itself with Problem 21 completed to your instructor, who will give you free response Problems 22-30. You can use your calculator for these problems, but cannot return to Problems 1-21.

Name: ______________ **Alpha** ________ **Instructor** ____________ **Section:** _______

CONTINUED FREE RESPONSE. The remaining 9 problems are not multiple choice. Answer them on this test paper in the blank space provided. If space is insufficient use the backs of the pages. Show the details of your work and $\boxed{\text{box}}$ your answers. You MAY use your calculator on these problems. Show all inputs that you enter into your calculator.

22. a) Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

$L_1: \quad x = 3t, \quad y = 2 - 2t, \quad z = -4 + t \qquad \langle 3,-2,1\rangle$ = NOT PARALLEL

$L_2: \quad x = -3 + 4s, \quad y = 4 - s, \quad z = -5 + 2s \qquad \langle 4,-1,2\rangle =$ /

$3t = -3+4s \qquad 3t = -3+8t+8 \Rightarrow -5t = 5 \Rightarrow \boxed{t=-1,\ s=0}$

$2-2t = 4-s \Rightarrow s = 2t+2$

will this hold true for "z"?

$\Rightarrow -4+t \stackrel{?}{=} -5+2s$

$\Rightarrow -4-1 \stackrel{?}{=} -5+2(0)$

$-5 = -5$ ✓✓

$\therefore$ $\boxed{\text{Intersect at } (-3,4,-5)}$

b) Determine the angle in degrees to five significant digits between the planes with equations:

$\langle 1,3,-1\rangle \leftarrow \vec{n}_1 \leftarrow x + 3y - z = 2$

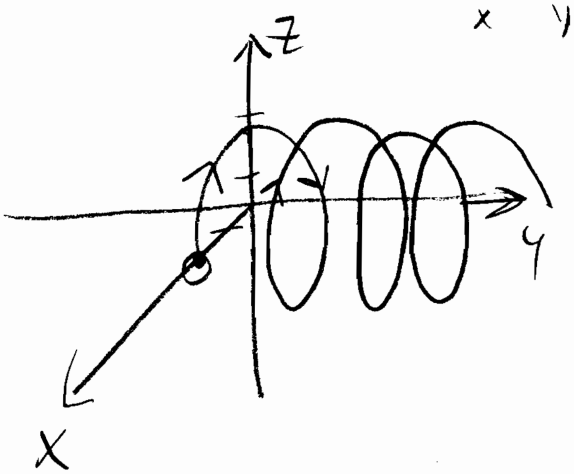
$\langle 2,-1,4\rangle \leftarrow \vec{n}_2 \leftarrow 2x - y + 4z = 0$

} angle between normals

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{2-3-4}{\sqrt{11}\sqrt{21}} = \boxed{109.21^\circ}$$

23. Sketch and describe the curve with the given vector equation. Include coordinate axes with tick marks at every unit and include an arrow on the curve in the direction in which parameter t increases: $\mathbf{r}(t) = \langle 2\cos(t), t, 2\sin(t)\rangle$

x y z



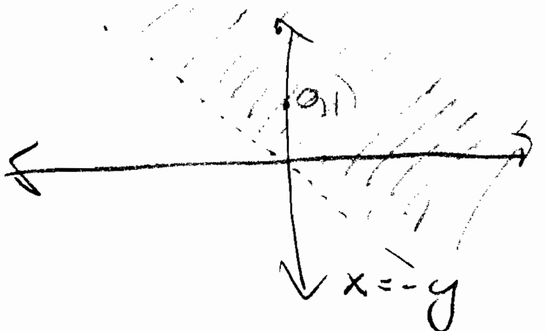
* helix
* radius = 2
* travels in y direction
* passes through $\langle 2,0,0\rangle$
*

Free Response CALCULATOR ALLOWED

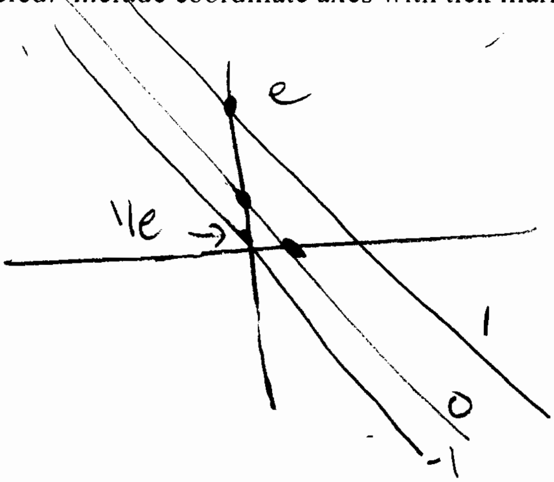
24. For $g(x, y) = \ln(x + y)$:

a) Find and sketch the domain of g.

$x + y > 0 \Rightarrow x > -y$



b) Sketch a contour map of g showing three level curves with the values of g on them labeled. Include coordinate axes with tick marks at every unit.



0 contour for $x+y=1$

$\Rightarrow y = -x+1$

1 contour for $x+y=e$

$\Rightarrow y = -x+e$

-1 contour for $x+y = 1/e$

$\Rightarrow y = -x + 1/e$

etc

25. Suppose the depth of the water at and near your location, $(x, y) = (3,5)$, is given by $z = 3 + 2x - xy + y^2$. In which direction will depth increase the fastest, and what is that maximal rate of increase?

$$\vec{\nabla} f = \left\langle \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right\rangle = \langle 2-y, -x+2y \rangle$$

$$\therefore \vec{\nabla} f(3,5) = \langle 2-5, -3+2(5) \rangle$$

$$= \boxed{\langle -3, 7 \rangle}$$

$$|\vec{\nabla} f| = \sqrt{9+49} = \boxed{\sqrt{58}}$$

Free Response CALCULATOR ALLOWED

26. Set up the iterated integral and evaluate to find $\iiint_E xz\,dV$ where E is the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,3,0)$, and $(0,0,4)$. (Note: the "slanting" face of E has equation $\frac{x}{1}+\frac{y}{3}+\frac{z}{4}=1$.) $\Rightarrow 12x+4y+3z=12$

$$\int_0^1\int_0^{3-3x}\int_0^{4-\frac{4}{3}y-4x} xz\,dz\,dy\,dx$$

$\Rightarrow z = 4-\frac{4}{3}y-4x$

when $z=0$

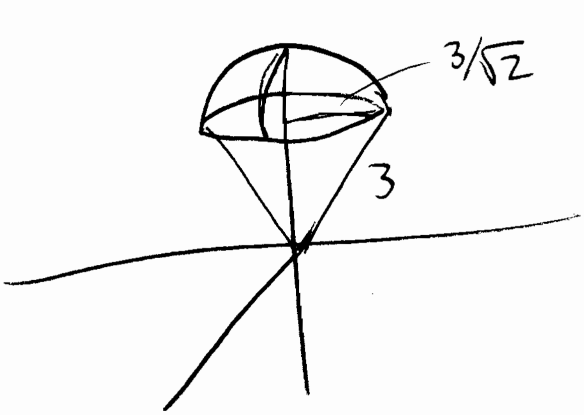
$\frac{4}{3}y = 4-4x \Rightarrow y=3-3x$

when $z,y=0$

$0\le x\le 1$

$= 2/5$

27. Find the volume of the solid E that lies above the cone $z=\sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2=9$. Sketch E.



$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^3 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

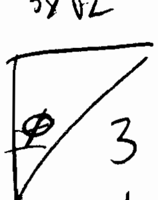
z^2

$\Rightarrow x^2+y^2+(x^2+y^2)=9$

$\Rightarrow 2(x^2+y^2)=9 \Rightarrow x^2+y^2=9/2$

$\Rightarrow r=3/\sqrt{2}$

finding intersection between sphere & cone so that we can find ϕ



$\sin\phi=\frac{1}{\sqrt{2}}$

$\phi=\pi/4$

$= 9(2-\sqrt{2})\pi$

≈ 16.56

Free Response CALCULATOR ALLOWED

28. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ in the following two ways where $\mathbf{F}(x,y,z) = 2xy\mathbf{i} + x^2\mathbf{j} + \mathbf{k}$ and C is the line segment from $(0,0,0)$ to $(3,2,1)$:

a) By parametcrizing C and integrating directly.

$\vec{r}(t) = \langle 3t, 2t, t\rangle \qquad \vec{F}(\vec{r}(t)) = \langle 12t^2, 9t^2, 1\rangle$

$\vec{r}\,'(t) = \langle 3, 2, 1\rangle \qquad \vec{F}\cdot r'(t) = 36t^2 + 18t^2 + 1$

$= 54t^2 + 1 \Rightarrow \int_0^1 (54t^2 + 1)\,dt = 18t^3 + t\Big|_0^1 = 19$

b) By finding a potential function f with $\nabla f = \mathbf{F}$ and using the Fundamental Theorem for Line Integrals.

$\left.\begin{aligned} \int 2xy\,dx &= x^2y \\ \int x^2\,dy &= x^2y \\ \int dz &= z \end{aligned}\right\} \quad f(x,y,z) = x^2y + z$

$\int \vec{F}\,dr \Rightarrow f(3,2,1) - f(0,0,0)$

$= 18 + 1 - 0 \quad 19$

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29. Evaluate (directly) the surface integral $\iint_S \mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F}(x,y,z) = xy\mathbf{i} + x^2\mathbf{j} + yz\mathbf{k}$ and S is the part of the surface with upward orientation and equation $z = xe^y$ that lies above the square $0 \le x \le 1$ and $0 \le y \le 1$.

$xe^y \qquad \vec{n} = \langle -e^y, -xe^y, 1\rangle$

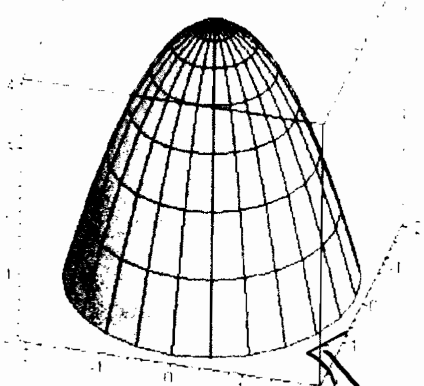
$\iint \vec{F}\cdot\vec{n}\,dA = \int_0^1\int_0^1 \langle xy, x^2, y\underbrace{z}_{xye^y}\rangle \cdot \langle -e^y, -xe^y, 1\rangle\,dx\,dy$

$= \int_0^1\int_0^1 (-\cancel{xye^y} - x^3e^y + \cancel{xye^y})\,dx\,dy$

$= \int_0^1\int_0^1 -x^3e^y\,dx\,dy = \int_0^1 -\frac{1}{4}x^4\Big|_0^1 e^y\,dy$

$= -\frac{1}{4}\Big|_0^1 e^y = \boxed{-\frac{1}{4}(e-1) \approx -.42957}$

30. Use the divergence theorem to calculate the surface integral $\iint_S \mathbf{F}\cdot d\mathbf{S}$, the flux of $\mathbf{F}$ across S, for $\mathbf{F}(x,y,z) = 2x^2\mathbf{i} + xy\mathbf{j} - z\mathbf{k}$ and the surface S that is the positively oriented boundary of the region bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.



$$\iiint \vec{\nabla}\cdot\vec{F}\,dV = \iint \vec{F}\cdot dS$$

$$\iiint (4x + x - 1)\,dV$$

$x^2+y^2=4$

$$= \iiint (5x-1)\,dV = \iiint (5r\cos(\theta)-1)\,r\,dr\,d\theta\,dz$$

$$= \int_0^{2\pi}\int_0^{2}\int_0^{4-r^2} (5r\cos(\theta)-1)\,r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{2}\int_0^{4-r^2} (5r^2\cos(\theta)-r)\,dz\,dr\,d\theta$$

$$= -8\pi$$