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1. **d.** $\vec{\mathbf{u}}\times\vec{\mathbf{v}}$, $\dfrac{\vec{\mathbf{u}}}{|\vec{\mathbf{u}}|}$ and $2\vec{\mathbf{u}}-\vec{\mathbf{v}}$ are vectors and $\vec{\mathbf{u}}\cdot|\vec{\mathbf{v}}|$ is not defined.

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2. **b.** An equation of the plane containing the point (x_0, y_0, z_0) normal to the vector $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$ is given by $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$.

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3. **c.** The graph is a cylinder. The edges are parallel to the x axis, so x is not a part of the equation. For any given value of y there are two values of z. So y is a function of z.

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4. **e.** The variable, ρ, in spherical coordinates is the distance from the origin therefore we have $x^2+y^2+z^2=\rho^2=4$ so $\rho=2$.

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5. **a.** c) is a circle of radius 2 traversed clockwise. b) and d) are a circle or radius 1 traversed twice and e) is a spiral.

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6. **c)** $\vec{\mathbf{v}}(t)=\vec{\mathbf{r}}'(t)=\langle 1,-\sin(t),\cos(t)\rangle$. The speed is the magnitude of the velocity and is equal to $\sqrt{1+\sin^2(t)+\cos^2(t)}=\sqrt{2}$.

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7. **b.** $W_v(-15,20)\approx\dfrac{W(015,30)-W(-15,20)}{30-20}=\dfrac{-26-(-24)}{10}=-2/10=-0.2$

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8. **d.** $g_y<0$ because as we move in the positive y direction the z values represented by the contours decrease. The contours spread out. Therefore g_y becomes less negative as we move in the positive y direction and therefore $g_{yy}>0$.

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9. **e.** $\dfrac{dz}{dt}=\dfrac{\partial z}{\partial x}\dfrac{dx}{dt}+\dfrac{\partial z}{\partial y}\dfrac{dy}{dt}$. $x(1)=1, y(1)=2$. $\dfrac{dx}{dt}=2t$, $\left.\dfrac{dx}{dt}\right|_{t=1}=2$, $\dfrac{dy}{dt}=2$. Therefore,

$\dfrac{dz}{dt}=4*2+1*2=10$.

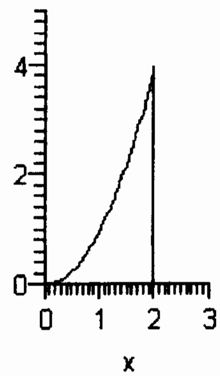
---

10. **a.** $\nabla f=2x^2\hat{\mathbf{i}}+2y\hat{\mathbf{j}}$, $\nabla f(2,-2)=12\hat{\mathbf{i}}-4\hat{\mathbf{j}}$. The unit vector is $\hat{\mathbf{u}}=\dfrac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}=\dfrac{4}{5}\hat{\mathbf{i}}+\dfrac{3}{5}\hat{\mathbf{j}}$.

Therefore, $D_u=(12\hat{\mathbf{i}}-4\hat{\mathbf{j}})\cdot\left(\dfrac{4}{5}\hat{\mathbf{i}}+\dfrac{3}{5}\hat{\mathbf{j}}\right)=\dfrac{48}{5}-\dfrac{12}{5}=\dfrac{36}{5}=7.2$.

---

11. **e**. The region is shown to the right. When we reverse the order of integration we have $0 \le y \le 4$ and $\sqrt{y} \le x \le 2$.



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12. **a**. $\int_0^{\pi/4}\int_0^2 r^2 r\,dr\,d\theta = \pi/16 \approx 0.19635$

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13. **e**. $\int_0^2\int_0^{x^3}\left(x^2+y^2\right)dydx$ or $\int_0^2\int_0^{x^3}\int_0^{x^2+y^2} dzdydx = \frac{224}{5} = 44.8$

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14. **c**. $\int_0^{2\pi}\int_0^2\int_0^{4-r^2} zrdzdrd\theta = \frac{32\pi}{3} \approx 33.5103$

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15. **c**. $\int_0^{2\pi}\int_0^{\pi/4}\int_0^2 \rho^2 \sin(\phi)d\rho\, d\varphi\, d\theta = \frac{-8\left(\sqrt{2}-2\right)\pi}{3} \approx 4.90747$.

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16. **c**. $dy=2x$. $\int_0^2\left(x^2\right)^2 dx + x * 2xdx = \int_0^2 x^4 + 2x^2 dx = \frac{176}{15} \approx 11.7333$

---

17. **c**. $\frac{\partial P}{\partial y} = x^2$, $\frac{\partial Q}{\partial x} = 3kx^2$. To be conservative, they must be equal, so $k=1/3$.

---

18. **d**. By Green's theorem,

$$\oint_C ydx + x^2dy = \oint_C Pdx + qdy = \iint_D \frac{\partial Q}{dx} - \frac{\partial P}{dy}dA = \int_0^3\int_{x^2}^9 (2x-1)dydx.$$

---

19. **b**. The arrows moving away from the point are longer than the arrows approaching the point. Therefore the divergence is positive. The arrows above the point are longer than those below so there is a clockwise spin. Therefore the curl is non-zero.

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20. **d**. The surface is of the form $z=g(x,y)$ and the integrand is a scalar function.

$$\iint_S f(x,y,z)dS = \iint_D f(x,y,g(x,y))\sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2 + 1}\,dA$$

$$= \int_0^2\int_0^3 x\left(4-x^2\right)\sqrt{4x^2+1}\,dydx \approx 28.76444$$

---

21. a) $4\langle 3,-4,12\rangle - 2\langle 1,2,-2\rangle = \langle 12-2,-16-4,48+4\rangle = \langle 10,-20,52\rangle$

b) $\cos(\theta) = \dfrac{\vec{\mathbf{u}}\cdot\vec{\mathbf{v}}}{|\vec{\mathbf{u}}||\vec{\mathbf{v}}|} = \dfrac{3-8-24}{\sqrt{9+16+144}\sqrt{1+4+4}} = \dfrac{-29}{13*3} = \dfrac{-29}{39}$,

$\theta = \cos^{-1}(-29/39) \approx 2.40922\text{rad}$.

c) $comp_{\vec{\mathbf{u}}}(\vec{\mathbf{v}}) = \dfrac{\vec{\mathbf{u}}\cdot\vec{\mathbf{v}}}{|\vec{\mathbf{u}}|} = \dfrac{-29}{13} \approx -2.23077$

d) $\dfrac{\vec{\mathbf{u}}\times\vec{\mathbf{v}}}{|\vec{\mathbf{u}}\times\vec{\mathbf{v}}|} = \dfrac{\langle -16,18,10\rangle}{2\sqrt{170}} \approx \langle -.613572,.690168,.383482\rangle$.

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22. a) $\overrightarrow{PQ} = \langle -1,1,0\rangle$. $x=1-t$, $y=1+t$, $z=2$.

b) $4=1-t$. $t=-3$, $y=-2$, $z=2$. The point is (4,-2,2).

c) $\overrightarrow{PR} = \langle 4,-2,-4,\ \rangle$ $\overrightarrow{PQ}\times\overrightarrow{PR} = \langle -4,-4,-2\rangle$. Therefore the equation of the plane is $-4(x-1)-4(y-1)-2(z-2)=0$ or $2x+2y+z=6$.

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23. a) *i)* $\vec{\mathbf{v}}(t) = \vec{\mathbf{r}}'(t) = -4\sin(t)\hat{\mathbf{i}} + 4\cos(t)\hat{\mathbf{j}}$, $\vec{\mathbf{r}}'(2\pi) = 4\hat{\mathbf{j}}$.

ii) $|\vec{\mathbf{r}}'(2\pi) = 4|$. *iii)* $\vec{\mathbf{r}}''(t) = -4\cos(t)\hat{\mathbf{i}} - 4\sin(t)\hat{\mathbf{j}}$, $\vec{\mathbf{r}}''(2\pi) = -4\hat{\mathbf{i}}$.

b) When the string breaks we have $\vec{\mathbf{r}}(2\pi) = 4\hat{\mathbf{i}} + 2\hat{\mathbf{k}}$. After the string breaks, the acceleration is $-9.8\hat{\mathbf{k}}$. If we restart the clock the equations of motion are then $x(t)=4$, $y(t)=4t$, $z(t)=-4.9t^2+2$. The ball hits the ground after $t=\sqrt{\dfrac{2}{4.9}} \approx .638877$ sec and the point where it hits is about (4,2.5555,0) feet measured from the feet of the Midshipman.

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24. a) D_T indicates that under the current conditions, the take off distance will increase 7.7 feet for every degree increase in the temperature. D_A indicates that under the current conditions the take off distance will increase 0.14 foot for every one foot increase in the altitude.

b) $D(T,A) \approx 1530 + 7.7(T-30) + 0.14(A-4000)$.

c) $D(33,4500) \approx 1530 + 7.7(3) + 0.14(500) \approx 1623$ feet.

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25. a) $f_x(x,y) = 3x^2y - y^4, \quad f_x(1,2) = -10$

b) $f_y(x,y) = x^3 - 4xy^3, \quad f_y(1,2) = -31$

c) $\vec{\nabla} f(1,2) = -10\hat{\mathbf{i}} - 31\hat{\mathbf{j}}$

d) $\dfrac{\vec{\nabla} f}{\left|\vec{\nabla} f\right|} = \dfrac{-10\hat{\mathbf{i}} - 31\hat{\mathbf{j}}}{\sqrt{1061}}$.

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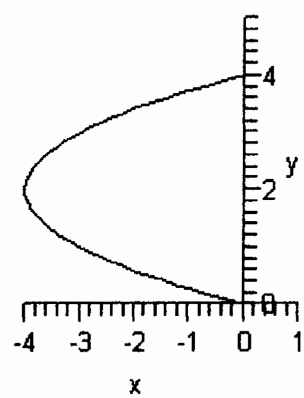
26. The region is shown to the right.. The integral is then

$\int_0^4 \int_{y^2-4y}^0 x^3y - y^4 \, dxdy \approx -598.146$.



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27. $m = \int_0^3 \int_0^x \int_0^{x+y} dzdydx = 27/2 \qquad \bar{x} = 2/27 \int_0^3 \int_0^x \int_0^{x+y} xdzdydx = 9/4$

$\bar{y} = 2/27 \int_0^3 \int_0^x \int_0^{x+y} ydzdydx = 5/4 \qquad \bar{z} = 2/27 \int_0^3 \int_0^x \int_0^{x+y} zdzdydx = 7/4$

Therefore the centroid is (9/4,5/4,7/4).

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28. a) $curl(\vec{\mathbf{F}}) = \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle = \left\langle 0-0, 3z^2 - 3z^2, 2x - 2x \right\rangle = \vec{\mathbf{0}}$.

b) $f(x,y,z) = \int (2xy + z^3)dx = x^2 y + xz^3 + g1(y,z)$. $\frac{\partial f}{\partial y} = x^2 + \frac{\partial g1}{\partial y} = x^2 + 2y$.

Therefore $\frac{\partial g1}{\partial y} = 2y$ and $g1(y,z) = y^2 + g2(z)$. We now have

$f(x,y,z) = x^2 y + xz^3 + y^2 + g2(z)$ We get $\frac{\partial f}{\partial z} = 3xz^2 + g2'(z) = 3xz^2$ Therefore we finally have $f(x,y,z) = x^2 y + xz^3 + y^2 + c$.

c) $x^2 y + xz^3 + y^2 \Big|_{(2,1,1)}^{(4,5,6)} = 965$. 962

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29. $\vec{\mathbf{r}}(\theta, z) = \langle 2\cos(\theta), 2\sin(\theta), z \rangle$, $0 \le \theta \le 2\pi$, $0 \le z \le 3$. $\vec{\mathbf{r}}_\theta = \langle -2\sin(\theta), 2\cos(\theta), 0 \rangle$, $\vec{\mathbf{r}}_z = \langle 0,0,1 \rangle$. $\vec{\mathbf{r}}_\theta \times \vec{\mathbf{r}}_z = \langle 2\cos(\theta), 2\sin(\theta), 0 \rangle$. $\vec{\mathbf{F}}(\vec{\mathbf{r}}(\theta,z)) = \langle 2\cos(\theta), 2\sin(\theta), 0 \rangle$.
$\vec{\mathbf{F}} \cdot (\vec{\mathbf{r}}_\theta \times \vec{\mathbf{r}}_z) = 4\cos^2(\theta) + 4\sin^2(\theta) = 4$.
Therefore $\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}} = \int_0^{2\pi} \int_0^3 4\, dz\, d\theta = 24\pi \approx 75.3982$.

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30. Since the region is a sphere, we will integrate in spherical coordinates.
$\nabla \cdot \vec{\mathbf{F}} = 3x^2 + 3y^2 + 3z^2 = 3\rho^2$. $\int_0^{2\pi} \int_0^{\pi} \int_0^1 3\rho^2 \rho^2 \sin(\phi) d\rho\, d\phi\, d\theta = 12\pi / 5 \approx 7.53982$.

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31. From Stokes' theorem $\iint_S curl(\vec{\mathbf{F}}) \cdot d\vec{\mathbf{S}} = \int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$. For this surface, the bounding curve, C is a circle of radius 2 in the plane z=4. This curve can be paratrized by $\vec{\mathbf{r}}(t) = \langle 2\cos(t), 2\sin(t), 4 \rangle$, $0 \le t \le 2\pi$. We therefore have $d\vec{\mathbf{r}} = \langle -2\sin(t), 2\cos(t), 0 \rangle dt$.
$\vec{\mathbf{F}}(\vec{\mathbf{r}}(t)) = 2\sin(t)\hat{\mathbf{i}} - 2\cos(t)\hat{\mathbf{j}} + 16\hat{\mathbf{k}}$ and $\vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = (-4\sin^2(t) - 4\cos^2(t))dt = -4dt$. We then have $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_0^{2\pi} -4dt = -8\pi \approx -25.1527$.
