
You may use a calculator on all problems. Write your name, alpha code, and section on your blue book(s) and the bubble sheet. Bubble in your alpha code in the left most columns of the bubble sheet.

Part one. Multiple Choice (50%). The first 20 problems are multiple choice. Fill in the letter of the best answer on the bubble sheet. There is no penalty for a wrong answer. YOU MUST ALSO WRITE YOUR ANSWERS AND SHOW ALL WORK IN YOUR BLUE BOOK(S).

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1. If $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are vectors, which of the following is a scalar?

a) $\vec{\mathbf{u}}\times\vec{\mathbf{v}}$ b) $\dfrac{\vec{\mathbf{u}}}{|\vec{\mathbf{u}}|}$ c) $\vec{\mathbf{u}}\cdot|\vec{\mathbf{v}}|$ d) $\dfrac{\vec{\mathbf{u}}\cdot\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$ e) $2\vec{\mathbf{u}}-\vec{\mathbf{v}}$

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2. An equation of the plane containing the point (1,-1,5) normal to the vector $\vec{\mathbf{n}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ is

a) $1(x-3)-1(y-4)+5(z+2)=0$ b) $3(x-1)+4(y+1)-2(z-5)=0$

c) $x-y+5z=0$ d) $x=3t+1,\ y=4t-1,\ z=4t+5$

e) $x=1t-3,\ y=4t-2.\ z=-2t+4$

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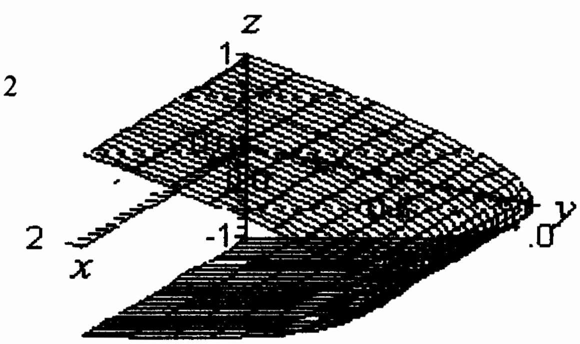
3. The surface shown has the equation

a) $x=1-z^2$ b) $y=1-x^2$ c) $y=1-z^2$

d) $z=1-x^2$ e) $z=1-y^2$



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4. The equation $x^2+y^2+z^2=4$ is equivalent to

a) $r=2$ b) $\theta=2$ c) $z=2$ d) $\phi=2$ e) $\rho=2$

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5. A circle of radius 2 in the *xy* plane centered at the origin traversed counter clockwise may have the vector equation with $0\le t\le 2\pi$

a) $\vec{\mathbf{r}}(t)=\langle 2\cos(t), 2\sin(t)\rangle$ b) $\vec{\mathbf{r}}(t)=\langle \cos(2t), \sin(2t)\rangle$ c) $\vec{\mathbf{r}}(t)=\langle 2\sin(t), 2\cos(t)\rangle$

d) $\vec{\mathbf{r}}(t)=\langle \sin(2t), \cos(2t)\rangle$ e) $\vec{\mathbf{r}}(t)=\langle t\cos(t), t\sin(t)\rangle$

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6. If the position of a particle is given by $\vec{\mathbf{r}}(t)=\langle t, \cos(t), \sin(t)\rangle$ then the *speed* at $t=0$ is

a) $\langle 1,-\sin(t),\cos(t)\rangle$ b) $\langle 1,0,1\rangle$ c) $\sqrt{2}$ d) $\langle 0,-\cos(t),-\sin(t)\rangle$ e) 1

T/v	**20**	**30**	**40**
-10	-18	-20	-21
-15	-24	-26	-27
-20	-30	-33	-34

7. The table shows the wind-chill index, $W(T,V)$ as a function of the actual temperature, T (°C) and the wind speed, v(km/hr). Then the numerical value of $W_v(-15,20)$ is approximately

a) -24 b) -0.2 c) 0.6 d) -15 e) 20

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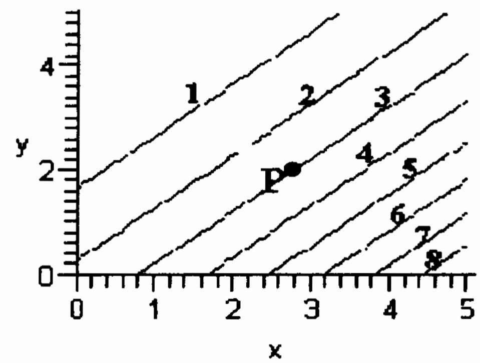
8. Several contours of the function $g(x,y)$ are shown, At the point, P,



a) $g_y = 0, g_{yy} = 0$ b) $g_y > 0, g_{yy} = 0$

c) $g_y > 0, g_{yy} < 0$ d) $g_y < 0, g_{yy} > 0$

e) $g_y < 0, g_{yy} < 0$

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9. If $z = f(x,y)$, $\left.\frac{\partial f}{\partial x}\right|_{(1,2)} = 4$, $\left.\frac{\partial f}{\partial y}\right|_{(1,2)} = 1$, $x = t^2, y = 2t$ then $\left.\frac{dz}{dt}\right|_{t=1}$ is closest to

a) 2 b) 4 c) 6 d) 8 e) 10

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10 For the function, $f(x,y) = x^3 + y^2$, at the point (2,-2) the directional derivative in the direction of the vector $\vec{\mathbf{v}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ is

a) 7.2 b) 36 c) $\frac{12}{5}x^2 - \frac{6}{5}y$ d) $\left\langle \frac{48}{5}, \frac{-12}{5} \right\rangle$ e) $\left\langle \frac{12x^2}{5}, \frac{-6y}{5} \right\rangle$

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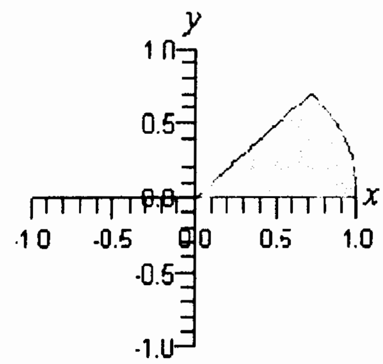
11. Reversing the order of integration in the iterated integral $\int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx$ gives

a) $\int_0^{x^2} \int_0^2 f(x,y)\,dx\,dy$ b) $\int_0^2 \int_0^{\sqrt{y}} f(x,y)\,dx\,dy$ c) $\int_0^2 \int_{\sqrt{y}}^2 f(x,y)\,dx\,dy$

d) $\int_0^4 \int_0^{\sqrt{y}} f(x,y)\,dx\,dy$ e) $\int_0^4 \int_{\sqrt{y}}^2 f(x,y)\,dx\,dy$

12. The mass of a lamina bounded by $y=0$, $y=x$ inside the circle $x^2+y^2=1$ (see picture) with a mass density of $\rho(x,y)=x^2+y^2$ is closest to

a) 0.2 b) 0.3 c) 0.4 d) 1.0 e) 1.6



13. The volume of the solid bounded above by the surface $z=x^2+y^2$ and below by the portion of the xy plane bounded by $x=0$, $x=2$, $y=0$, $y=x^3$ is closest to

a) 5 b) 15 c) 25 d) 35 e) 45

14. The mass of the solid bounded by $z=4-x^2-y^2$, and the xy plane with mass density given by $\rho(x,y,z)=z$ is closest to

a) 15 b) 25 c) 34 d) 54 e) 96

15. The volume of the portion of the sphere of radius 2 centered at the origin that is above the cone $z=\sqrt{x^2+y^2}$ is closest to

a) 0 b) 2.5 c) 5.0 d) 7.5 e) 10.0

16. $\int_C y^2dx+xdy$ along the curve $y=x^2, 0\le x\le 2$ is closest to

a) 0 b) 6 c) 12 d) 18 e) 24

17. If the vector field $\langle x^2y, kx^3\rangle$ is conservative then $k=$

a) -3 b) -1/3 c) 1/3 d) 1 e) 3

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18. From Green's theorem we have that $\oint_C ydx + x^2dy$ along the positively oriented curve, C, which is the boundary of the region enclosed by $x = 0,\ y = x^2,\ y = 9$ is equal to

a) $\int_0^3\int_0^{x^2} (2x-1)dydx$ b) $\int_0^3\int_0^{x^2} (1-2x)dydx$ c) $\int_0^3\int_0^{x^2} (2x+1)dydx$

d) $\int_0^3\int_{x^2}^9 (2x-1)dydx$ e) $\int_0^3\int_{x^2}^9 (1-2x)dydx$

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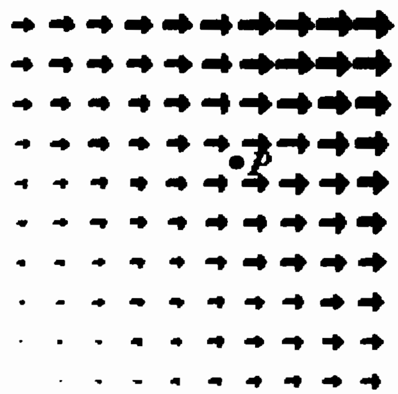
19. The vector field, $\vec{\mathbf{F}}(x,y,z)$ is shown to the right. (Only the portion in the xy plane is shown. The vector field looks the same on all other planes parallel to the xy plane and the third component is 0).
Which of these statements is true at the point P?



a) The curl is not $\vec{0}$ and the divergence is negative.
b) The curl is not $\vec{0}$ and the divergence is positive
c) The curl is $\vec{0}$ and the divergence is negative
d) The curl is $\vec{0}$ and the divergence is 0
e) The curl is $\vec{0}$ and the divergence is positive

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20. $\iint_S xzdS$ where S is the portion of the surface, $z = 4 - x^2,\ 0 \le x \le 2,\ 0 \le y \le 3$ is closest to

a) -0.5   b) 0   c) 12   d) 29   e) 35

Part 2. Longer Answers(50%). Work 10 of the following 11 problems. SHOW ALL WORK AND YOUR ANSWERS IN YOUR BLUE BOOK(S).

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21. Given the vectors $\vec{\mathbf{u}} = \langle 3,-4,12\rangle$ and $\vec{\mathbf{v}} = \langle 1,2,-2\rangle$ find

a) $4\vec{\mathbf{u}} - 2\vec{\mathbf{v}}$
b) The angle between the two vectors
c) $comp_{\vec{\mathbf{u}}}(\vec{\mathbf{v}})$
d) A unit vector orthogonal to both $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$

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22. Given the points $P(1,1,2)$, $Q(0,2,2)$ and $R(5,-1,-2)$, find
a) parametric equations of the line though the points P and Q.
b) The point where the line in part a) intersects the plane $x=4$.
c) An equation of the plane containing the points P, Q and R.

23. A ball is being swung in a horizontal circle of radius 4 meters about the head of a Midshipman at a height of 2 meters. The ball's position is given by
$r(t) = 4\cos(t)\hat{\mathbf{i}} + 4\sin(t)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$.
a) Find at $t = 2\pi$ i) the velocity, ii) the speed, iii) the acceleration.
b) At time $t = 2\pi$ the rope breaks and the ball now moves under ballistic motion with acceleration due to gravity of $\vec{\mathbf{a}}(t) = -9.8\hat{\mathbf{k}}m/\sec^2$. i) How long after the string breaks does the ball hit the ground? Where does it hit the ground?

24. The normal take off distance, D, of an A6 which weighs 35000lbs when there is a head wind of 20 knots is a function of the temperature, T, in degrees F; altitude, A, in feet. A table of this distance shows that if T=30° and A=4000ft then $D(30,4000)$=1530ft. Furthermore we have that $D_T(30,4000) \approx 7.7 ft/\deg$ and $D_A(30,4000) \approx 0.14 ft/ft$.
a) Explain in words the meaning of the derivatives.
b) Find a linear approximation for the take off distance for values of temperature and altitude near (30,4000)
c) Use the linear approximation to estimate $D(33,4500)$.

25. Given the function, $f(x,y) = x^3y - xy^4$,
a) Find $f_x(1,2)$
b) Find $f_y(1,2)$
c) Find the gradient, $\vec{\nabla} f(1,2)$
d) Find a unit vector in the direction of greatest increase of the function, $f(x,y)$ at the point (1,2)

26. Find $\iint_D x^3y - y^4 dA$ where D is the region bounded by $x = y^2 - 4y$ and $x = 0$.

27. Find the mass and the centroid (density equals 1) of the solid bounded above by the surface $z = x + y$ and below by the triangle in the xy plane, $0 \le x \le 3$, $0 \le y \le x$.

28. Given the vector field, $\vec{\mathbf{F}}(x,y,z)=\left\langle 2xy+z^3, x^2+2y, 3xz^2\right\rangle$

a) Compute the curl($\vec{\mathbf{F}}$).
b) If the vector field is conservative, find a scalar function, $f(x,y,z)$ such that $\vec{\mathbf{F}}(x,y,z)=grad(f)$.
c) Use your answer to part b) to find $\int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$ where C is a curve from (2,1,1) to (4,5,6).

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29. Compute $\iint_S \vec{\mathbf{F}}\cdot d\vec{\mathbf{S}}$ where $\vec{\mathbf{F}}(x,y,z)=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}$ and S is the side of the cylinder of radius 2 centered on the z axis oriented outward with $0\le z\le 3$.

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30. Use the divergence theorem to compute $\iint_S \vec{\mathbf{F}}\cdot d\vec{\mathbf{S}}$ where $\vec{\mathbf{F}}(x,y,z)=\left\langle x^3,y^3,z^3\right\rangle$ and S is the sphere $x^2+y^2+z^2=1$ oriented outward.

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31. Use Stokes' theorem to find $\iint_S curl(\vec{\mathbf{F}})d\vec{\mathbf{S}}$ where $\vec{\mathbf{F}}(x,y,z)=y\hat{\mathbf{i}}-x\hat{\mathbf{j}}+z^2\hat{\mathbf{k}}$ and S is the portion of the paraboloid $z=x^2+y^2$ with $0<z<4$ oriented upward.