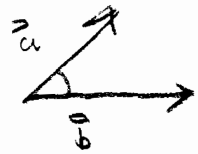
Multiple Choice

① 

$\text{proj}_{\vec b}\vec a = |\vec a|\cos\theta \frac{\vec b}{|\vec b|} = \cancel{|\vec a|}\frac{\vec a\cdot\vec b}{\cancel{|\vec a|}|\vec b|}\frac{\vec b}{|\vec b|} = \frac{\vec a\cdot\vec b}{|\vec b|^2}\vec b$

$\Rightarrow \frac{\langle 4,2,2\rangle\cdot\langle 1,2,2\rangle}{1^2+2^2+2^2}\langle 1,2,2\rangle = \frac{\cancel{12}\,4}{\cancel{9}\,3}\langle 1,2,2\rangle = \langle \frac{4}{3}, \frac{8}{3}, \frac{8}{3}\rangle$

(d)

② $\vec n = \langle 1,2,-3\rangle \Rightarrow x+2y-3z=d \Rightarrow -2+16-15=-1$

$\Rightarrow \boxed{x+2y-3z=-1}$ (b)

③ $\vec r\,'(t) = \langle 1, 2t, 3t^2\rangle \qquad \vec r\,'(1) = \langle 1,2,3\rangle$

$\Rightarrow T(1) = \frac{r'(1)}{|r'(1)|} = \frac{\langle 1,2,3\rangle}{\sqrt{14}}$ (b)

④ $F = mA \Rightarrow \vec a(t) = \vec r\,''(t)$

$\Rightarrow \vec r\,'(t) = \langle 3t^2, 2t, 3t^2\rangle, \quad \vec r\,''(t) = \langle 6t, 2, 6t\rangle$

$\Rightarrow F = m\vec a = (1)\vec a = \langle 6t, 2, 6t\rangle$ (c)

⑤ $s(t) = |v(t)| \Rightarrow s'(t) = \frac{|v(t+\Delta t)| - |v(t)|}{\Delta t}$ (e)

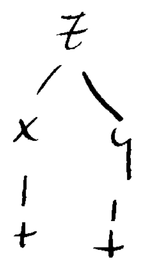
throw out

(6) $\frac{\partial u}{\partial x} = 2cxy$ $\qquad$ $\frac{\partial u}{\partial y} = cx^2 - 3y^2$

$\frac{\partial^2 u}{\partial x^2} = 2cy$ $\qquad$ $\frac{\partial^2 u}{\partial y^2} = -6y$

$\Rightarrow 2cy - 6y = 0 \Rightarrow c = 3$ $\qquad$ (a)

(7)



$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

$\left.\frac{\partial z}{\partial t}\right|_{t=3} = 6(5) + 8(4) = 62$ $\qquad$ (b)

(8) $2z^2 = 21 - x^2 - 3y^2 \Rightarrow z^2 = \frac{21}{2} - \frac{1}{2}x^2 - \frac{3}{2}y^2$

$z = \left(\frac{21}{2} - \frac{1}{2}x^2 - \frac{3}{2}y^2\right)^{1/2}$

$\Rightarrow \vec{r}(x,y) = \langle x, y, \left(\frac{21}{2} - \frac{1}{2}x^2 - \frac{3}{2}y^2\right)^{1/2}\rangle$

$\Rightarrow \vec{r}_x(x,y) = \langle 1, 0, -\frac{x}{2}\left(\frac{21}{2} - \frac{1}{2}x^2 - \frac{3}{2}y^2\right)^{-1/2}\rangle$

$\Rightarrow \vec{r}_y(x,y) = \langle 0, 1, -\frac{3y}{2}\left(\frac{21}{2} - \frac{1}{2}x^2 - \frac{3}{2}y^2\right)^{-1/2}\rangle$

$\Rightarrow \vec{r}_x(4,-1) = \langle 1, 0, -2\left(\frac{21}{2} - \frac{16}{2} - \frac{3}{2}\right)^{-1/2}\rangle = \langle 1, 0, -2\rangle$

$\vec{r}_y(4,-1) = \langle 1, 0, \frac{3}{2}\rangle$

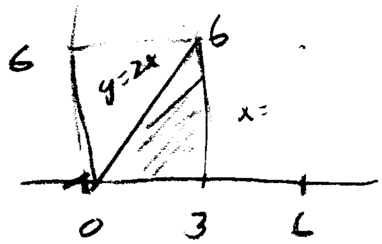
$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 1 & 3/2 \end{vmatrix} = \langle 2, -3/2, 1\rangle \Rightarrow 2x - \frac{3}{2}y + z = 8 + \frac{3}{2} + 1 = \frac{21}{2}$

$8x - 6y + 4z = 42$

(e)

⑨ $\nabla f = \langle 2x-2,\ 3y^2 \rangle$

$\nabla f(2,1) = \underline{\underline{\langle 2,3\rangle}} \Rightarrow \langle -2,-3\rangle$ ⓒ

⑩ 

$x = y/2 \Rightarrow y = 2x$

$$\int_0^3\int_0^{2x} \underline{e^{ax^2}}\,dy\,dx$$

$$\int_0^3 2xe^{ax^2}\,dx \qquad u = ax^2$$
$$du = 2ax\,dx \Rightarrow 2x\,dx = \frac{du}{a}$$

$$\Rightarrow \frac{1}{a}\int_0^3 e^u\,du = \frac{1}{a}e^u = \frac{1}{a}e^{ax^2}\Big|_0^3 = \frac{1}{a}\left(e^{9a}-1\right)$$

ⓐ

⑪ $\iint x+2y\,dA =$ 

$$\int_0^{\pi}\int_1^2 (r\cos\theta + 2r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_0^{\pi}\int_1^2 r^2(\cos\theta + 2\sin\theta)\,dr\,d\theta$$

$$= \int_0^{\pi} (\cos(\theta) + 2\sin\theta)\,d\theta \int_1^2 r^2\,dr$$

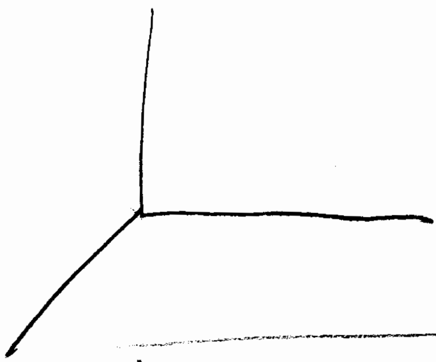
$$= \cancel{\sin\theta} - 2\cos(\theta)\Big|_0^{\pi} \quad \frac{1}{3}r^3\Big|_1^2 = -2((-1)-1)\left(\frac{8}{3}-\frac{1}{3}\right)$$

$\Rightarrow 0$

$$= 4\left(\frac{7}{3}\right) = 28/3$$

ⓐ

(12)



$z = 4 - 4x - 2y$

$z=0 \Rightarrow y = 2-2x$

$y, z = 0 \Rightarrow x = 1$

$$\int_0^1 \int_0^{2-2x} \int_0^{4-4x-2y} \delta \, dz\, dy$$

(d)

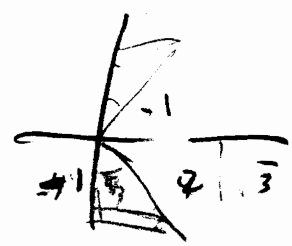
(13)

$$\rho = (x^2+y^2+z^2)^{1/2} = (0+12+4)^{1/2} = 4$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\infty = \pi/2$$

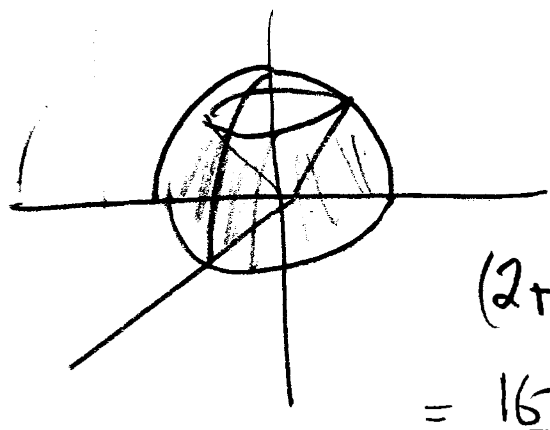
$$\phi = \cos^{-1}\frac{z}{\rho} = \frac{-2}{4} = \cos^{-1}(-1/2)$$

$$= 2\pi/3$$



(e)

(14)



$$\int_0^{2\pi}\int_{\pi/4}^{\pi}\int_0^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$(2\pi)\left(\tfrac{1}{3}\right)(2)^3 \quad \int_{\pi/4}^{\pi/2} \sin\phi \, d\phi$$

$$= \frac{16\pi}{3} \quad -\cos(\phi)\Big|_{\pi/4}^{\pi/2}$$

$$= \left(\frac{16\pi}{3}\right)\left(0 - \frac{\sqrt{2}}{2}\right) = \frac{8\sqrt{2}\pi}{3}$$

(b)

(15) $\frac{\partial Q}{\partial x} = 1 \qquad \frac{\partial P}{\partial y} = -1$ not conservative

① $\vec{r}(t) = \langle t, t^2 \rangle$, $0 \le t \le 1$ ② $\vec{F}(t) = \langle -t^2, t \rangle$

③ $\vec{r}\,'(t) = \langle 1, 2t \rangle$

$$\Rightarrow \int_0^1 \langle -t^2, t \rangle \cdot \langle 1, 2t \rangle \, dt = \int_0^1 -t^2 + 2t^2 \, dt$$

$$= \int_0^1 t^2 \, dt = \frac{1}{3}$$

(d)

(16) conservative force field

$$x^3y + y^2x \Big|_{(0,0)}^{(4,-1)}$$

$$= (64)(-1) + (1)(4) = -60$$

(a)

(17) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & x^2 & y^2 \end{vmatrix} = \hat{i}(2y) - \hat{j}(-2z) + \hat{k}(2x)$

$$= \langle 2y, +2z, 2x \rangle$$

(c)

(18) $\frac{\partial}{\partial x} f\vec{F} + \frac{\partial}{\partial y} f\vec{F} + \frac{\partial}{\partial z} f\vec{F}$

$$f_x F + f\vec{F}_x + f_y F + f\vec{F}_y + f_z \vec{F} + f\vec{F}_z$$

$$\Rightarrow \nabla f \cdot \vec{F} \cdot f\,\mathrm{div}(\vec{F})$$

(b)

(19) $\iint dS = \iint |\vec{n}| dA$

$$= \iint |\langle 2, 6, 2 \rangle| dA$$

$$\sqrt{44} \int_0^1 \int_0^2 dA = (2)\sqrt{44} = 4\sqrt{11}$$

(c)

(20) Divergence Theorem

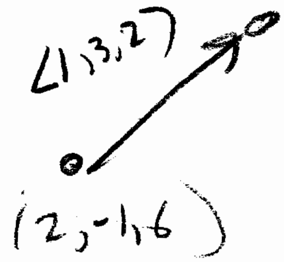
$$\iiint \operatorname{div}(\vec{F})\,dV = \iiint (1-2+2)\,dV$$

$$= \iiint dV = (1)(2)(3) = \underline{\underline{6}}$$

d

Long Answer

①



$\underline{\underline{(3,2,8)}}$

② a) $\boxed{\vec{r}(t) = \langle 1,2,0\rangle + \langle 1,-2,4\rangle t}$

b) $\boxed{\begin{aligned} x &= 1+t \\ y &= 2-2t \\ z &= 4t \end{aligned}}$

$$(1+t) - 2(2-2t) + 4(4t) = 18$$

$$1+t-4+4t+16t = 18$$

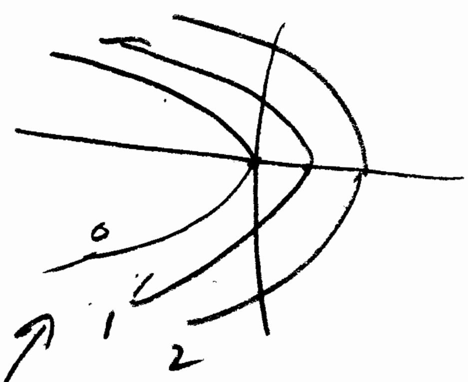
$$21t - 3 = 18 \Rightarrow t = 1$$

$$\Rightarrow \begin{aligned} x &= 2 \\ y &= 0 \\ z &= 4 \end{aligned}$$

$$\boxed{(2,0,4)}$$

③

0	$0 = x+y^2$	$\Rightarrow$	$x = -y^2$
1	$1 = x+y^2$		$x = 1-y^2$
2	$2 = x+y^2$		$x = 2-y^2$



stacked parabolas

④ $D_u f = \nabla f \cdot \vec{u}$

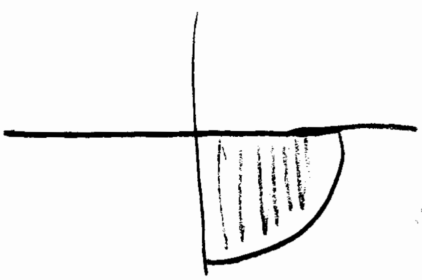
$\Rightarrow \langle f_x, f_y \rangle \cdot \langle 1/\sqrt{5}, 2/\sqrt{5} \rangle$

$= \langle 3, f_y \rangle \cdot \langle \frac{1}{\sqrt{5}}, 2/\sqrt{5} \rangle = 2$

$= \frac{3}{\sqrt{5}} + \frac{2 f_y}{\sqrt{5}} = 2 \Rightarrow 3 + 2f_y = 2\sqrt{5}$

$\Rightarrow \boxed{f_y = \frac{2\sqrt{5}-3}{2}}$ ✓

⑤



$\iint (x^2+y^2)^{a/2} \, dA$

$0 \le r \le 1$

$\frac{3\pi}{2} \le \theta \le 2\pi$

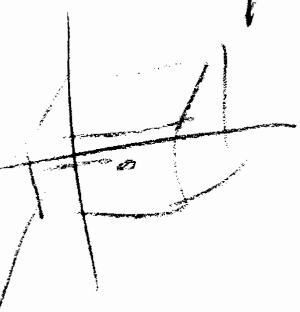
$= \int_{3\pi/2}^{2\pi} \int_0^1 (r^2)^{a/2} \, r \, dr \, d\theta$

$= \int_{3\pi/2}^{2\pi} \int_0^1 r^{a+1} \, dr \, d\theta$

$= \left(\frac{\pi}{2}\right) \frac{1}{a+2} r^{a+2} \Big|_0^1 = \boxed{\frac{\pi}{2(a+2)}}$ ✓

(6) $m = \int_0^1\int_0^2\int_0^3 z\,dz\,dx\,dy = (1)(2)\left(\tfrac{1}{2}z^2\right)\Big|_0^3 = 9$

$M_{yz} = \int_0^1\int_0^2\int_0^3 x z\,dz\,dy\,dx$



$\int_0^1\int_0^2 \frac{9x}{2}\,dy\,dx = \int_0^1 9x\,dx = 9/2$

$\Rightarrow M_{yz}/m = \bar{x} \Rightarrow \bar{x} = 1/2$

(7) $\int(2xyz^2+3)\,dx = x^2yz^2 + 3x \ldots$

$\int(x^2z^2+3z+1)\,dy = x^2yz^2 + 3yz + y \ldots$

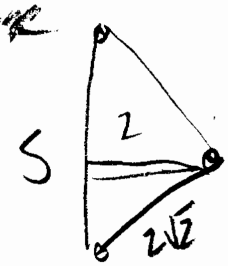
$\int(2x^2yz+3y+2)\,dz \quad x^2yz^2 + 3yz + 2z \ldots$

$$\boxed{f(x,y,z) = x^2yz^2 + 3x + 3yz + y + 2z + C}$$

(8) Greens Theorem

area of triangle

$$\iint (1-2)\,dA = -\iint dA$$



$$-\tfrac{1}{2}(5)(2) = -5$$

Throw out

#9 & #10