0755 – 1055 Tuesday 12 December 2006

You may use a calculator on all problems. Write your name, alpha code and section number on your blue book(s) and the bubble sheet. **Bubble in your alpha code in the left most columns of the bubble sheet.**

Part One is Multiple Choice. The first 20 questions are multiple choice. Fill in the letter of the correct answer on the bubble sheet. There is no penalty for a wrong answer. Depending upon your instructor, you may also have to show all your work for these problems in your blue books.

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1. The vector projection of $\vec{v} = \langle 4,2,2 \rangle$ onto $\vec{w} = \langle 1,2,2 \rangle$ is

a) 4/3 b) 12 c) $\langle 2,1,1 \rangle$ d) $\langle 4/3, 8/3, 8/3 \rangle$ e) $\langle 1/2,1,1 \rangle$

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2. An equation for the plane through the point $(-2,8,5)$ and perpendicular to the line $x = 1+t$, $y = 3+2t$, $z = 4-3t$ is

a) $x+3y+4z = 43$ b) $x+2y-3z = -1$ c) $(1+t)x+(3+2t)y+(4-3t)z = 43$
d) $(x-1)^2 + (y-3)^2 + (z-4)^2 = 35$ e) $\vec{r}(t) = \langle -2,8,5 \rangle + t\langle 1,1,1 \rangle$

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3. If $\vec{r}(t) = \langle t, t^2, t^3 \rangle$, then the *unit* tangent vector at $t = 1$, $\vec{T}(1)$, is

a) $\langle 1,2,3 \rangle$ b) $\langle 1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14} \rangle$ c) $\langle 0,2,6 \rangle$ d) $\langle 0, 2/\sqrt{40}, 6/\sqrt{40} \rangle$
e) $\langle 1/\sqrt{3}, 1/\sqrt{3}, 1/3 \rangle$

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4. The force required so that a particle of mass m = 1 has the position function $\vec{r}(t) = \langle t^3, t^2, t^3 \rangle$ is

a) $\vec{F} = \langle t^4/4, t^3/3, t^4/4 \rangle$ b) $\vec{F} = \langle t^5/20, t^4/12, t^5/20 \rangle$ c) $\vec{F} = \langle 6t, 2, 6t \rangle$
d) $\vec{F} = \langle t^3, t^2, t^3 \rangle$
e) not enough information is given to determine the force.

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5. Suppose at time t the position of a particle is $\vec{r}(t)$, the velocity is $\vec{v}(t)$, the acceleration is $\vec{a}(t)$, and the speed is $s(t)$. Then $s'(1)$ equals

a) $\vec{r}(1) \times \vec{v}(1)$ b) $\vec{a}(1)$ c) the component of $\vec{v}(1)$ in the direction of $\vec{a}(1)$ d) $\vec{v}(1) \times \vec{a}(1)$
e) the component of $\vec{a}(1)$ in the direction of $\vec{v}(1)$

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6. The value of c for which $u(x,y) = cx^2y - y^3$ is a solution to $\frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y} = 0$ is

a) 3 b) π c) 2 d) 0 e) $-\pi$

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7. Let $z = f(x,y)$, where f is differentiable. Suppose $x = g(t)$, $y = h(t)$ and $g(3) = 2$, $g'(3) = 5$, $h(3) = 7$ and $h'(3) = 4$. Also suppose $f_x(2,7) = 6$ and $f_y(2,7) = 8$. Then when $t = 3$ the derivative $\dfrac{dz}{dt}$ equals

a) 68 b) 62 c) 23 d) 32 e) 0

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8. An equation for the tangent plane of the surface $x^2 + 3y^2 + 2z^2 = 21$ at the point (4,-1,1) is

a) $2x^2 + 6y^2 + 4z^2 = 42$ b) $x + 3y + 2z = 3$ c) $\vec{r}(t) = \langle 4,-1,1\rangle + t\langle 2x,6y,4z\rangle$
d) $\vec{r}(t) = \langle 4,-1,1\rangle + t\langle 8,-6,4\rangle$ e) $8x - 6y + 4z = 42$

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9. The direction of greatest *__decrease__* of the function $f(x,y) = x^2 - 2x + y^3$ at the point (2,1) is

a) $-\sqrt{13}$ b) $\sqrt{13}$ c) $\langle -2,-3\rangle$ d) $\langle -3,2\rangle$ e) $\langle -3/\sqrt{13}, 2/\sqrt{13}\rangle$

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10. Reversing the order of integration in the integral $\int_0^6 \int_{y/2}^3 e^{ax^2}\,dx\,dy$ and then integrating yields which of the following as the exact value of the integral for $a \neq 0$?

a) $\dfrac{e^{9a}-1}{a}$ b) $\dfrac{e^{8a}-1}{a}$ c) $6(e^{9a} - e^{ax^2/4})$ d) $6(e^{9a} - e^{ay^2/4})$ e) $\dfrac{4}{3a}$

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11. Suppose R is the planar region which lies above the x-axis between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. Then $\iint_R (x + 2y)\,dA$ equals

a) $\dfrac{28}{3}$ b) 0 c) $\dfrac{-7\pi}{3}$ d) $\dfrac{15}{2}$ e) $\dfrac{15}{4}$

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12. Consider the solid bounded by the coordinate planes and the plane $4x + 2y + z = 4$. If the mass density of the solid is $\delta(x,y,z)$, then an integral that always gives the total mass of the solid is

a) $\int_0^2\int_0^3\int_0^6 \delta(x,y,z)\,dz\,dy\,dx$ b) $\int_0^2\int_0^3\int_0^6 x\delta(x,y,z)\,dz\,dy\,dx$ c) $\int_0^{4-2y-4x}\int_0^{2-2x}\int_0^2 \delta(x,y,z)\,dx\,dy\,dz$

d) $\int_0^1\int_0^{2-2x}\int_0^{4-2y-4x} \delta(x,y,z)\,dz\,dy\,dx$ e) $\int_0^{\pi/2}\int_0^{\pi/2}\int_0^6 \delta(\rho,\theta,\phi)\rho^2\sin(\phi)\,d\rho\,d\theta\,d\phi$

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13. If the rectangular coordinates of a point are $(0,2\sqrt{3},-2)$, the spherical coordinates (ρ,θ,ϕ) are

a) $(4,\pi/2,\pi/3)$ b) $(4,0,\pi/3)$ c) $(16,\pi,\pi/3)$ d) $(16,0,\pi)$ e) $(4,\pi/2,2\pi/3)$

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14. The volume of the solid that lies within the sphere $x^2+y^2+z^2=4$, above the xy plane and below the cone $z=\sqrt{x^2+y^2}$ is

a) $16\pi/3$ b) $8\sqrt{2}\pi/3$ c) 4π d) $4\sqrt{2}\pi/3$ e) $4\sqrt{2}\pi$

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15. The work done by the force $\vec{F}=-y\hat{i}+x\hat{j}$ on a particle which moves along the curve $\vec{r}(t)=\langle t,t^2\rangle$ for $0\le t\le 1$ is

a) 1 b) $-1/3\hat{i}+1/2\hat{j}$ c) 0 d) 1/3 e) $-\hat{i}+\hat{j}$

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16. If $f(x,y)=x^3y+y^2x$, then $\nabla f=\langle 3x^2y+y^2,x^3+2xy\rangle$. The line integral $\int_C \langle 3x^2y+y^2,x^3+2xy\rangle \bullet d\vec{r}$ for C the path consisting of the line segments from $(0,0)$ to $(1,1)$, from $(1,1)$ to $(3,1)$, from $(3,1)$ to $(4,0)$, and from $(4,0)$ to $(4,-1)$ equals

a) -60 b) 0 c) $\langle -47,56\rangle$ d) $\langle 56,-47\rangle$ e) 3

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17. The curl of $\vec{F}=\langle z^2,x^2,y^2\rangle$ is

a) $2x+2y+2z$ b) $\langle 2z,2y,2z\rangle$ c) $\langle 2y,2z,2x\rangle$ d) 0 e) $\vec{0}$

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18. If f is a differentiable scalar function and $\vec{F}$ is a differentiable vector field, then $div(f\vec{F})$ always equals

a) 0 b) $\nabla f\bullet\vec{F}+f\,div(\vec{F})$ c) $f\,div(\vec{F})$ d) $div(\nabla f\times\vec{F})$ e) $\nabla f\bullet curl(\vec{F})$

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19. The surface area of the portion of the plane $2x+6y+2z=17$ which lies above the rectangle $0\le x\le 1$, $0\le y\le 2$ is

a) $3\sqrt{12}$ b) $2\sqrt{12}$ c) $4\sqrt{11}$ d) $2\sqrt{15}$ e) $2\sqrt{11}$

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20. Let $\vec{F}=\langle x,-2y,2z\rangle$ and S the surface of the box $0\le x\le 1$, $1\le y\le 3$, $0\le z\le 3$ Then $\oiint_S \vec{F}\bullet d\vec{S}$ equals

a) -2 b) 0 c) 2π d) 6 e) $4\pi/3$

Part 2 Partial Credit. You may receive partial credit for the following questions. **Show all your work and answers in your blue books.**

1. If the vector $\vec{v} = \langle 1,3,2 \rangle$ is placed so that its tail is at the point $(2,-1,6)$, then at what point is its head?

2. *a)* Find an equation for the line which is perpendicular to the plane $x - 2y + 4z = 18$ which goes through the point $(1,2,0)$.
   b*)* Where does this line intersect the plane $x - 2y + 4z = 18$?

3. Sketch three level curves for $f(x,y) = x + y^2$. Label the curves as to what values of f, they correspond.

4. Suppose $f(x,y)$ is differentiable and $f_x(1,1) = 3$ and $D_{\vec{u}} f(1,1) = 2$ for $\vec{u} = \langle 1/\sqrt{5}, 2/\sqrt{5} \rangle$. What is the value of $f_y(1,1)$?

5. Let R be the quarter disk $x^2 + y^2 \le 1$ that is in the fourth quadrant (i.e. $0 \le x$ and $y \le 0$). Compute $\iint (x^2 + y^2)^{a/2} dA$.

6. Let D be the box $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$. Suppose the mass density of D is $\delta(x,y,z) = z$. Find the x-coordinate of the center of mass of D.

7. The vector field $\vec{F} = (2xyz^2 + 3)\hat{i} + (x^2z^2 + 3z + 1)\hat{j} + (2x^2yz + 3y + 2)\hat{k}$ is conservative. Find a potential f for $\vec{F}$.

8. Let C be the triangle with vertices (0,0), (2,2) and (0,5) oriented counterclockwise. Compute $\int_C 2y\,dx + x\,dy$.

9. Let $\vec{F} = \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{(x^2 + y^2 + z^2)^{3/2}}$.

a) Compute $\iint_S \vec{F} \bullet d\vec{S}$ for S the outwardly oriented sphere $x^2 + y^2 + z^2 = R^2$.

b) Note: $div(\vec{F}(x, y, z)) = 0$ for $(x, y, z) \neq (0,0,0)$. Use this fact to compute $\iint_S \vec{F} \bullet d\vec{S}$ for any closed surface S which bounds a solid which contains the point (0,0,0) and which is oriented outward.

10. Consider the surface, S, with boundary curves C_1, C_2, C_3 - all oriented as shown. Suppose $\vec{F}$ is a vector field with the properties: *i*) $curl(\vec{F}) = \vec{0}$ on S, *ii*) $\int_{C_1} \vec{F} \bullet d\vec{r} = 3$, and *iii*) $\int_{C_2} \vec{F} \bullet d\vec{r} = 2$. What does $\int_{C_3} \vec{F} \bullet d\vec{r}$? Why?